Homework 4

Due date: April 28, 2017

Problem 1:

(a) Find a linear mapping $G(u, v)$ that maps the unit square $[0, 1] \times [0, 1]$ to the parallelogram in the xy-plane spanned by the vectors $\langle 3, -1\rangle$ and $\langle 1, 4\rangle$.

(b) Use the Jacobian to find the area of the image of the rectangle $\mathcal{R} = [0, 4] \times [0, 3]$ under G.

Problem 2: Use the change of variables $x = u^2 - v^2$, $y = 2uv$ to evaluate the integral

$$\iint_{\mathcal{A}} y \, dA$$

where $\mathcal{A}$ is the region bounded by the x-axis and the parabolas $y^2 = 4 - 4x$ and $y^2 = 4 + 4x$, with $y \geq 0$. The uv-integration domain will be chosen in the first quadrant.

Problem 3: Let $\mathcal{D}$ be the region in the first quadrant of the xy-plane bounded by the curves

$$y = \frac{2}{x} \quad , \quad y = \frac{1}{2x} \quad , \quad y = 2x \quad , \quad y = \frac{x}{2}$$

and F the map from he xy-plane to the uv-plane given by $u = xy$ and $v = \dfrac{y}{x}$.

(a) Sketch $\mathcal{D}$ and find the image of $\mathcal{D}$ under F.

(b) Let $G = F^{-1}$. Determine $|\mathrm{Jac}(G)|$.

(c) Apply the Change of Variables formula to find a relation between

$$\iint_{\mathcal{D}} f\left(\frac{y}{x}\right) dA \qquad \text{and} \qquad \int_{\frac{1}{2}}^{2} \frac{f(v)}{v} \, dv.$$

(d) Use (c) to evaluate $\displaystyle\iint_{\mathcal{D}} \frac{y e^{\frac{y}{x}}}{x} \, dA$.