Homework 1

Elements of solution

Problem 1: Let $f(x,y) = e^{xy} \cdot x$. Find the partial derivatives f_x, f_y and f_{xy}.

First,

$$f_x = y \cdot e^{xy} \cdot x + e^{xy}$$

and

$$f_y = e^{xy} \cdot x^2.$$

Then

$$f_{xy} = y \cdot e^{xy} \cdot x^2 + e^{xy} \cdot 2x.$$

Problem 2: Use the Fundamental Theorem of Calculus to find the derivatives of the following functions:

1. $f(x) = \int_1^{3x+2} \frac{t}{1+t^3} dt$**;**

2. $g(x) = \int_{\sin x}^{1} \sqrt{1+t^2} dt$**.**

For the first one, by the Chain Rule

$$f'(x) = \frac{3x+2}{1+(3x+2)^3} \cdot 3.$$

And for the second one, we rewrite

$$g(x) = -\int_1^{\sin x} \sqrt{1+t^2} dt.$$

By Chain Rule again, we have that

$$g'(x) = -\sqrt{1+\sin^2 x} \cdot \cos x.$$

Problem 3: Find $f(x)$ if $f'(x) = x\sqrt{x}$ and $f(1) = 2$.

Let

$$f(x) = \int_0^x t\sqrt{t}dt = \int_0^x t^{\frac{3}{2}}dt = \frac{2}{5}x^{\frac{5}{2}} + C$$

Because $f(1) = 2$, we conclude that

$$f(1) = \frac{2}{5} + C = 2.$$

Thus $C = \frac{8}{5}$ and

$$f(x) = \frac{2}{5}x^{\frac{5}{2}} + \frac{8}{5}.$$