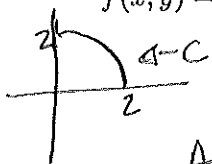
Math 13 Worksheet #9: Line integrals

(1) Find the area of a wall whose base is the part of the circle with radius 2 centered at the origin, lying in the first quadrant, and whose height at point (x, y) is given by $f(x, y) = 2x + y$.



C: $x = 2\cos t \quad y = 2\sin t \quad 0 \le t \le \pi/2$

$$A = \int_C f(x,y)\,ds = \int_0^{\pi/2} f(x,y)\,|\bar{r}'(t)|\,dt$$

$$\boxed{|\bar{r}'(t)| = \sqrt{4\cos^2 t + 4\sin^2 t} = 2}$$

$$= \int_0^{\pi/2} (4\cos t + 2\sin t)(2)\,dt$$

$$= 2\left[4\sin t - 2\cos t \,\Big|_0^{\pi/2}\right] = 2[4 - 2(0) - 0 + 2]$$

$$= 2(6) = 12$$

(2) Find $\int_C (x + y + z)ds$, where C is the line segment from $(1, 4, 2\sqrt{3})$ to $(3, 7, 4\sqrt{3})$.

$0 < t < 1$

eqn for line segment $\bar{r}(t) = (1-t)\langle 1, 4, 2\sqrt{3}\rangle + t\langle 3, 7, 4\sqrt{3}\rangle$

$$= \langle 1 - t + 3t,\ 4 - 4t + 7t,\ 2\sqrt{3} - 2\sqrt{3}t + 4\sqrt{3}t\rangle$$

$$= \langle 1 + 2t,\ 4 + 3t,\ 2\sqrt{3} + 2\sqrt{3}t\rangle$$

$$|\bar{r}'(t)| = \sqrt{4 + 9 + 4\cdot 3} = \sqrt{13 + 12} = \sqrt{26}$$

$$\sqrt{26}\int_0^1 (1 + 2t + 4 + 3t + 2\sqrt{3} + 2\sqrt{3}t)\,dt = \sqrt{26}\int_0^1 (5 + 2\sqrt{3}) + (5 + 2\sqrt{3})t\,dt$$

$$= \sqrt{26}\ (5 + 2\sqrt{3})t + \frac{(5 + 2\sqrt{3})t^2}{2}\Big|_0^1$$

$$\boxed{= \sqrt{26}\,(5 + 2\sqrt{3})(3/2)}$$

(3) Evaluate $\int_C xye^{yz}dy$, where C: $x = t$, $y = t^2$, $z = t^3$, $0 \le t \le 1$

$$\int_C xye^{yz}dy = \int_0^1 t(t^2)e^{t^5}\,y'(t)\,dt = \int_0^1 t^3 e^{t^5}(2t)\,dt$$

$$= 2\int_0^1 t^4 e^{t^5}\,dt \qquad u = t^5 \quad du = 5t^4\,dt$$

$$= \frac{2}{5}\int_0^1 e^u\,du = \frac{2}{5}(e - 1)$$