Math 13 Worksheet #7: Vectors, dot product, cross product, and planes

(1) Set up the equation to find the angle between the vectors $\overline{PQ}$ and $\overline{PR}$ with $P(3,-1,2)$, $Q(8,2,4)$, and $R(1,-2,-3)$.

$\overline{PQ} = \langle 8-3,\ 2--1,\ 4-2\rangle = \langle 5,3,2\rangle$ $\quad |\overline{PQ}| = \sqrt{25+9+4} = 6$

$\overline{PR} = \langle 1-3,\ -2--1,\ -3-2\rangle = \langle -2,-1,-5\rangle$ $\quad |\overline{PR}| = \sqrt{4+1+25} = \sqrt{30}$

TWO WAYS

$\overline{PQ} \times \overline{PR} = |\overline{PQ}|\,|\overline{PR}|\sin\theta$ $\quad \theta$ is the angle inbetween $\overline{PQ}$ & $\overline{PR}$.

OR $\overline{PQ}\cdot\overline{PR} = |\overline{PQ}|\,|\overline{PR}|\cos\theta$

(2) Compute $\overline{PQ}\times\overline{PR}$. Geometrically what is the result?

$$\bar{n} = \overline{PQ}\times\overline{PR} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 5 & 3 & 2 \\ -2 & -1 & -5 \end{vmatrix} = \bar{i}\begin{vmatrix} 3 & 2 \\ -1 & -5\end{vmatrix} - \bar{j}\begin{vmatrix} 5 & 2 \\ -2 & -5\end{vmatrix} + \bar{k}\begin{vmatrix} 5 & 3 \\ -2 & -1\end{vmatrix}$$

$$= \langle -15+2,\ -(-25+4),\ -5+6\rangle = \langle -13, 21, 1\rangle$$

$\bar{n}$ is a vector $\perp$ to both $\overline{PQ}$ & $\overline{PR}$

(3) Find the equation of the plane through P and perpendicular to the vector $< 1,-2,5 >$.

$P(3,-1,2)$

Take the normal vector to be $\bar{n} = \langle 1,-2,5\rangle$ since it is perpendicular.

Then plane is given by

$\bar{n}\cdot(\langle x,y,z\rangle - P) = 0$

$\bar{n}\cdot\langle x-3,\ y+1,\ z-2\rangle = 0$

$\boxed{1(x-3) - 2(y+1) + 5(z-2) = 0}$