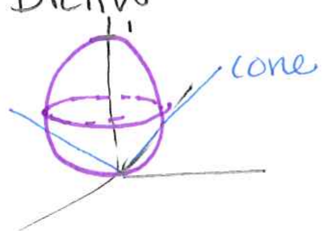
Math 13 Worksheet #6: Cylindrical and spherical coordinates

(1) (last problem from Worksheet #5)Find the volume of the solid that lies between the paraboloid $z = x^2 + y^2$ and the sphere $x^2 + y^2 + z^2 = 4$

See Worksheet5 solns.

(2) Use spherical coordinates to find the volume of the solid that lies about the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = z$.

Step 1: DRAW



STEP2: TRANSFORM surfaces.

cone: $z = \rho\cos\phi = \sqrt{x^2+y^2} = \rho\sin\phi$

$\Rightarrow \tan\phi = 1 \Rightarrow \phi = \pi/4$

sphere: $\rho^2 = \rho\cos\phi \Rightarrow \rho = \cos\phi$

STEP3: figure out bounds. $0 \le \theta \le 2\pi$ obvious.

sphere $\Rightarrow \rho$ in terms of ϕ $\Rightarrow$ integrate with respect to ρ first. $0 \le \rho \le \cos\phi$

Now bounds on ϕ. $0 \le \phi \le \pi/4$

STEP4: Set up Integral.

$$V = \int_0^{2\pi}\int_0^{\pi/4}\int_0^{\cos\phi} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

$$= \int_0^{2\pi}\int_0^{\pi/4} \frac{\rho^3}{3}\Big|_0^{\cos\phi} \sin\phi \, d\phi \, d\theta$$

$$= \int_0^{2\pi}\int_0^{\pi/4} \frac{\cos^3\phi}{3} \sin\phi \, d\phi \, d\theta = \int_0^{2\pi} -\frac{\cos^4\phi}{12}\Big|_0^{\pi/4} d\theta = \frac{\pi}{8}$$