Math 13 Worksheet #2: Double integrals over general regions

(1) Evaluate the double integral

$$\int_0^1 \int_0^{e^v} \sqrt{1+e^v}\, dw dv.$$

1st integrate wrt w

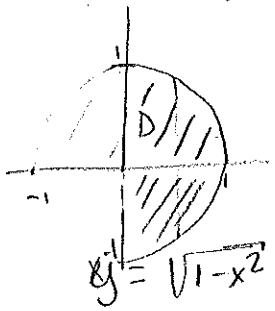
$$\int_0^1 \sqrt{1+e^v}\,(w|_0^{e^v})\, dv = \int_0^1 e^v \sqrt{1+e^v}\, dv$$

Use u-sub. $u = 1+e^v$, $du = e^v dv$

$$= \int_2^{e+1} \sqrt{u}\, du = \frac{2u^{3/2}}{3}\Big|_2^{e+1} = \frac{2}{3}\left((e+1)^{3/2} - 2^{3/2}\right)$$

(2) Evaluate the double integral $\iint_D xy^2 dA$ where D is the region enclosed by $x = 0$ and $x = \sqrt{1-y^2}$.

1st plot region. Chose Type I integral.



Type I: $\int_0^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} x y^2\, dy\, dx$

Type II: $\int_{-1}^1 \int_0^{\sqrt{1-y^2}} x y^2\, dx\, dy.$

I will integrate Type II.

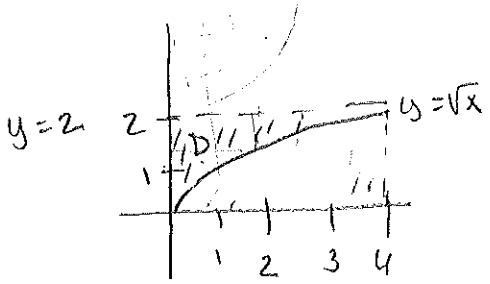
$$\text{Type II} = 2\int_0^1 \int_0^{\sqrt{1-y^2}} y^2 x\, dx dy = 2\int_0^1 \frac{x^2}{2}\Big|_0^{\sqrt{1-y^2}} y^2\, dy = \int_0^1 y^2 - y^4\, dy$$

$$= \frac{y^3}{3} - \frac{y^5}{5}\Big|_0^1 = 1/3 - 1/5 = \frac{5-3}{15} = 2/15$$

(3) Evaluate the integral by reversing the order of integration.

$$\int_0^4 \int_{\sqrt{x}}^2 \frac{1}{y^3+1} dy dx$$

Draw Integration region:



Now change limits. so integrating wrt x 1st. $\Rightarrow$ x goes from $0 \to y^2$

& y goes from $0 \to 2$

$$\Rightarrow \int_0^2 \int_0^{y^2} \frac{1}{y^3+1}\, dx dy$$

usob $u = y^3+1$, $du = 3y^2 dy$

$$= \int_0^2 \frac{x}{y^3+1}\Big|_0^{y^2} dy = \int_0^2 \frac{y^2}{y^3+1} dy$$

$$= \frac{1}{3}\int_1^9 \frac{1}{u} du = \frac{1}{3} \ln u\Big|_1^9 = \frac{1}{3} \ln 9$$