Math 13 Worksheet #19: Stokes' Theorem

(1) Verify Stokes' Theorem for the vector field $\boldsymbol{F}(x, y, z) =< -y, x, e^z >$ on the surface defined by $S = \{(x, y, z) : z = 1 - x^2 - y^2,\ x^2 + y^2 \leq 1\}$, with outward unit normal vector.

(2) Use Stokes' Theorem to evaluate to evaluate the integral of the vector field $\boldsymbol{F}(x, y, z) =< e^{xyz}, -xy^2z, xyz^2 >$ around the curve C given by $z^2 + y^2 = 9$ in the plane $x = 5$ and transversed in the counterclockwise direction when viewed from the right (i.e. where $x > 5$.)

(3) Evaluate $\iint_S \text{curl}\boldsymbol{F} \cdot \boldsymbol{n}dS$, where S is the cap of the unit sphere that lies below the $xy-$plane and inside the cylinder $x^2+y^2=\frac{1}{9}$ with outwards-pointing normal vector and where $\boldsymbol{F}(x,y,z) =< -yz^2, xz^2, 3^{-xyz} >$.

(4) For each of the following problems explain why Stokes' Theorem does not apply.

(a) S is the pyramid with vertices at $(0,0,6)$, $(2,0,0)$, $(-2,0,0)$, $(0,3,0)$, and $(0,-3,0)$.

(b) $\boldsymbol{F}(x,y,z) =< \ln(xy+1) + 5^x3^y2^z, 4xz^2 >$, and C is the boundary of the square in the plane $z=6$ and with vertices $(2,0,6)$, $(-2,0,6)$, $(2,4,6)$, and $(-2,4,6)$.