Math 13 Worksheet #12: Line integrals

(1) Give the vector field that is being integrated.

$$\int_C xy^2 dx + (xy - z)dy + \cos y dz$$

$\bar{F} = \langle xy^2, xy - z, \cos y \rangle$

(2) Compute $\int_C \boldsymbol{F} \cdot d\boldsymbol{r}$ where $\boldsymbol{F}(x, y, z) = \langle yz, x, z^2 \rangle$ with C the straight line segment from the origin to $(1, 0, 4)$.

$\bar{r} = (1-t)\langle 0,0,0\rangle + t\langle 1,0,4\rangle = \langle t, 0, 4t\rangle \quad 0 \le t \le 1$

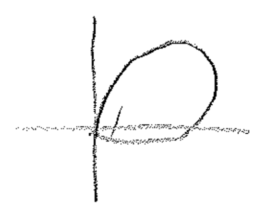
$$\int_C \bar{F} \cdot d\bar{r} = \int_0^1 \langle 0, t, 16t^2\rangle \cdot \langle 1, 0, 4\rangle \, dt$$

$$= \int_0^1 0 + 0 + 32t^2 \, dt = \frac{32t^3}{3}\Big|_0^1 = \frac{32}{3}$$

(3) Compute $\int_C \boldsymbol{F} \cdot d\boldsymbol{r}$ where $\boldsymbol{F}(x, y) = \langle ye^{xy}, xe^{xy} \rangle$ with C the cardioid $r = 1 + \sin(2\,\theta)$ from $\theta = -\pi/4$ to $\theta = 3\pi/4$.

$\bar{F}$ is conservative $\quad f(x,y) = e^{xy}$

C is a closed simple curve



$x = r\cos\theta$

$y = r\sin\theta$

$\Rightarrow$ ~~by~~ $\int_C \bar{F} \cdot d\bar{r} = 0.$

(4) Compute $\int_C \boldsymbol{F} \cdot dr$ where $\boldsymbol{F}(x, y) =< 2z + y, x, 2z + 2x >$ with C the curve $r(t) =< t^2, t, 3t >$, $1 \leq t \leq 2$.

$\bar{F}$ is conservative $\quad f(x,y,z) = 2zx + yx + z^2 + k.$

$$\int_C F \cdot dr = \int_1^2 \nabla f \cdot d\bar{r} = f(\bar{r}(2)) - f(\bar{r}(1))$$

$$= f(4, 2, 6) - f(1, 1, 3)$$

$$= 2(4)(6) + 4(2) + 36 - (6 + 1 + 1)$$