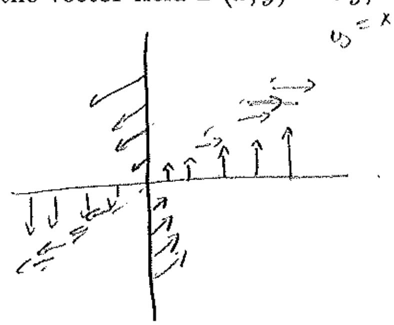
Math 13 Worksheet #10: Vector fields and work

(1) Draw the vector field $\boldsymbol{F}(x, y) =< y, x - y >$.



x	y	$\bar{F}(x,y)$
0	y	$<y, -y>$
x	0	$<0, x>$
x	x	$<x, 0>$

(2) Is the vector field $\boldsymbol{F}(x, y) =< 2xy, x^2 + 1 >$ conservative? If so, find the corresponding potential function f.

$P = 2xy \quad Q = x^2+1$

$P_y = 2x = Q_x = 2x$ ✓ $\Rightarrow$ conservative.

$\Rightarrow f_x = 2xy \rightarrow f(x,y) = x^2y + g(y)$

$f_y = x^2 + g'(y) = x^2+1$

$g'(y) = 1 \Rightarrow g(y) = y + C$ a-constant

$\rightarrow f(x,y) = x^2y + y + C$

(3) Find the gradient vector field of $f(x, y, z) = x \ln(y - 2z)$.

$F = \nabla f = < \ln(y-2z), \frac{x}{y-2z}, \frac{-2x}{y-2z} >$

(4) Evaluate $\int_C \boldsymbol{F} \cdot d\boldsymbol{r}$ where $\boldsymbol{F}(x, y, z) =< x,\ y,\ xy >$, $\boldsymbol{r}(t) =< \cos t,\ \sin t,\ t >$, and $0 \leq t \leq \pi$.

$\int_C F \cdot dr = \int_0^\pi F(\bar{r}(t)) \cdot \bar{r}'(t)\, dt$

$= \int_0^\pi <\cos t, \sin t, \cos t \sin t> \cdot <-\sin t, \cos t, 1> dt$

$= \int_0^\pi -\cos t \sin t + \cos t \sin t + \cos t \sin t\, dt$

$= \int_0^\pi \cos t \sin t\, dt = \int_0^\pi \frac{1}{2} \sin(2t)\, dt$

$= -\frac{1}{4} \cos(2t)\Big|_0^\pi$

$= -1/4(\cos(2\pi) - \cos(0)) = 0.$