Math 13: Written Homework # 6
Due May 7 at 5pm

Please make sure your homework is stapled, if necessary before handing it in. Do not use paper clips or any variation of folding techniques to connect papers.

Solutions should be justified in a rigorous way. If you are unsure how much work to show, you can ask me prior to turning in your assignment. The problems are taken from the 7th edition of Stewart's *Calculus*, although occasionally a problem will be modified to be slightly different from its textbook counterpart.

(1) (Problem #2, Chapter 16.4) Evaluate the line integral below by using two methods: Direct evaluation and the Green's Theorem. Verify that the answers are identical.

$$\int_C xydx + x^3dy,$$

where C is the rectangle (with positice orientation) with vertices $(0, 0)$, $(3, 0)$, $(3, 1)$ and $(0, 1)$.

(2) (Problem #22, Chapter 16.4) Let D be a region bounded by a simple closed path C in the $xy-$plane. Use Green's Theorem to prove that the coordiantes $(\bar{x}, \bar{y})$ of the centroid (the centroid is the center of mass of D, if we assume that D is a lamina of uniform density) of D are

$$\bar{x} = \frac{1}{2A}\int_C x^2dy, \quad \bar{y} = -\frac{1}{2A}\int_C y^2dx.$$

(3) (Problem #12, Chapter 16.5) Let f be a scalar function and $\boldsymbol{F} : \mathbb{R}^3 \to \mathbb{R}^3$ be a vector field. State whether each expression below is meaningful. If not, explain why. If so, state whether it is a scalar function or a vector field, and give a brief explanation why.

(a) curl f
(b) grad f
(c) div $\boldsymbol{F}$
(d) curl(grad f)
(e) grad $\boldsymbol{F}$
(f) grad(div $\boldsymbol{F}$)
(g) div(grad f)
(h) grad(div f)
(i) curl(curl $\boldsymbol{F}$)
(j) div(div $\boldsymbol{F}$)
(k) grad(f) × (div $\boldsymbol{F}$)
(l) div(curl(grad(f)))

(4) (Problem #20, Chapter 16.5) Is there a smooth vector field $\boldsymbol{G}$ on $\mathbb{R}^3$ such that $\nabla \times \boldsymbol{G} =< xyz, -y^2z, yz^2 >$? Explain.