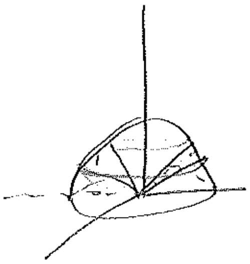
Homework#3

1)



$$V = \iiint_E dV = \int_0^{2\pi} \int_{\pi/6}^{\pi/3} \int_0^4 \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

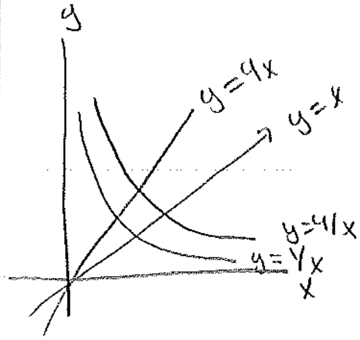
$$= \int_0^{2\pi} \int_{\pi/6}^{\pi/3} \frac{\rho^3}{3}\Big|_0^4 \sin\phi \, d\phi \, d\theta$$

$$= \frac{4^3}{3} \int_0^{2\pi} \underbrace{\int_{\pi/6}^{\pi/3} \sin\phi \, d\phi}\, d\theta$$

$$= \frac{4^3}{3}(2\pi) \quad -\cos\phi \Big|_{\pi/6}^{\pi/3} = \frac{4^3}{3}(2\pi)\left(-\cos(\pi/3) + \cos(\pi/6)\right)$$

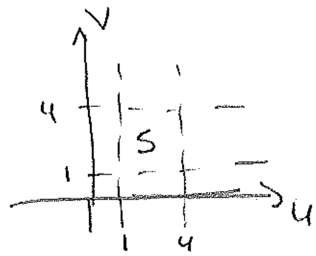
$$= \frac{2(4^3)}{3}\pi \left(\sqrt{3}/2 - 1/2\right)$$

$$= \frac{4^3}{3}\pi\left(\sqrt{3} - 1\right)$$

2) 

$\frac{y}{y} = 1 \qquad \frac{y}{x} = 4$

$xy = 4 \qquad yx = 1$



let $u = xy$, $v = y/x$ $\qquad 1 \le u \le 4 \qquad 1 \le v \le 4$

$y = vx \rightarrow u = x^2 v \rightarrow \boxed{x = \sqrt{u/v} \quad y = \sqrt{uv}}$

3) $\iint_R (x^2 - xy + y^2)\, dA \qquad R$ is ellipse $x^2 - xy + y^2 = 2$

$x = \sqrt{2}\, u - \sqrt{2/3}\, v$

$y = \sqrt{2}\, u + \sqrt{2/3}\, v$

$$J = \begin{vmatrix} \frac{dx}{du} & \frac{dx}{dv} \\ \frac{dy}{du} & \frac{dy}{dv} \end{vmatrix} = \begin{vmatrix} \sqrt{2} & -\sqrt{2/3} \\ \sqrt{2} & \sqrt{2/3} \end{vmatrix} = \frac{2}{\sqrt{3}} + \frac{2}{\sqrt{3}} = \frac{4}{\sqrt{3}}$$

$x^2 = (\sqrt{2}u - \sqrt{2/3}\,v)^2 = 2u^2 - 4/\sqrt{3}\, uv + 2/3\, v^2$

$y^2 = (\sqrt{2}u + \sqrt{2/3}\,v)^2 = 2u^2 + 4/\sqrt{3}\, uv + 2/3\, v^2$

$xy = (\sqrt{2}u - \sqrt{2/3}\,v)(\sqrt{2}u + \sqrt{2/3}\,v)$

$= 2u^2 - 2/3\, v^2$

$$\begin{aligned} x^2 - xy + y^2 = {} & 2u^2 - 4/\sqrt{3}\, uv + 2/3\, v^2 \\ & + 2u^2 + 4/\sqrt{3}\, uv + 2/3\, v^2 \\ & - 2u^2 \qquad\qquad + 2/3\, v^2 \\ \hline & 2u^2 + 6/3\, v^2 = 2u^2 + 2v^2 = 2 \end{aligned}$$

$$\underbrace{u^2 + v^2 = 1}_{S.}$$

$\iint_R (x^2 - xy + y^2)\, dA = \iint_S 2(u^2 + v^2)\, 4/\sqrt{3}\, dA$

$= 2\int_0^1 \int_0^{2\pi} \frac{4}{\sqrt{3}} r^3\, dr\, d\theta = \frac{2\cdot 4}{\sqrt{3}} \left.\frac{r^4}{4}\right|_0^1 2\pi = \frac{4\pi}{\sqrt{3}}$

R

4) $x-y=0 \qquad x+y=0 \qquad$ let $u=x-y$

$x-y=2 \qquad x+y=3 \qquad v=x+y$

$0\le u\le 2 \qquad 0\le v\le 3$

$J=$

$u+v=2x \Rightarrow x=\frac{1}{2}(u+v)$

$u-v=-2y \Rightarrow y=-\frac{1}{2}(u-v)$

$$J=\begin{vmatrix} 1/2 & 1/2 \\ -1/2 & 1/2 \end{vmatrix} = 1/4+1/4 = 1/2$$

$$\iint_R (x+y)e^{x^2-y^2}\,dA = \int_0^3\int_0^2 v e^{uv}\, 1/2\, du\, dv$$

$$=\frac{1}{2}\int_0^3 \frac{\cancel{v} e^{uv}}{\cancel{v}}\Big|_0^2 dv = \frac{1}{2}\int_0^3 (e^{2v}-1)\,dv$$

$$=1/2\left(1/2\, e^{2v} - v \Big|_0^3\right) = \frac{1}{4}e^6 - 3 - 1/4$$

$$= 1/4\, e^6 - 3\tfrac{1}{4}$$

5) normal vector for 1st plane $\bar{n}_1 = \langle 1,2,2\rangle$

normal vector for 2nd plane $\bar{n}_2 = \langle 2,-1,2\rangle$

The vectors are not parallel.

$\bar{n}_1\bar{n}_2 = 2-2+4 = 4 \neq 0. \Rightarrow$ not $\perp$.

$\Rightarrow$

$\bar{n}_1\cdot\bar{n}_2 = 4 = |\bar{n}_1||\bar{n}_2|\cos\theta \qquad |\bar{n}_1| = \sqrt{1+4+4} = 3$

$|\bar{n}_2| = \sqrt{4+1+4} = 3$

$\frac{4}{9} = \cos\theta$

$\theta = \arccos(4/9)$