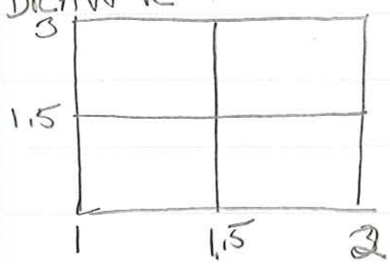
Math 13 HW#1

1) (a) Estimate the volume with height $z = 1 + x^2 + 3y = f(x,y)$
above $R = [1,2] \times [0,3]$ $\quad m = n = 2$

Soln: DRAW R



Sample pts are lower left corners
i.e. $(1,0), (1.5, 0), (1, 1.5), (1.5, 1.5)$

$\Delta A = 0.5 \cdot 1.5 = 0.75$

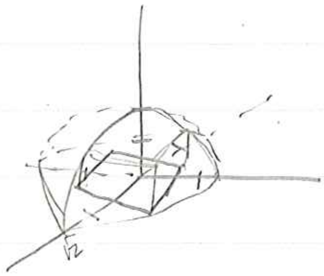
$V \sim \Delta A \left(f(1,0) + f(1.5,0) + f(1,1.5) + f(1.5,1.5) \right)$
$= 0.75(\quad \text{"} \quad)$
$= 14.625$

(b) midpt. $\Rightarrow$ Sample pts are
$(1.25, 3/4), (1.75, 3/4), (1.5, 2.25), (1.75, 2.25)$

$\Rightarrow V \sim 23.4375$

2) Sketch the solid. surface $z = 2 - x^2 - y^2$ [illegible]

$R = [0,1] \times [0,1]$



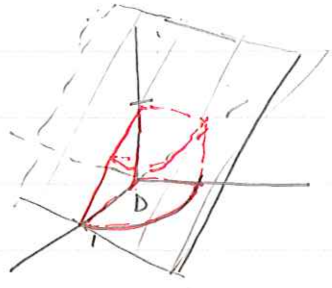
$$\int_0^1 \int_0^1 (2 - x^2 - y^2)\, dy\, dx$$

$$= \int_0^1 \left(2y - x^2 y - \frac{y^3}{3}\right)\Big|_0^1 dx$$

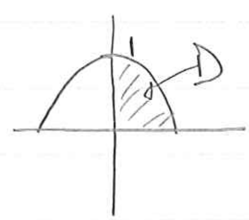
$$= \int_0^1 2 - x^2 - 1/3 \; dx = 2x - \frac{x^3}{3} - 1/3\, x \Big|_0^1$$

$$= 2 - 1/3 - 1/3 = 2 - 2/3 = 1\tfrac{1}{3}$$

3) Sketch the solid $z = 1 - x$



$D =$



$$\int_0^1 \int_0^{1-x^2} 1 - x \; dy\, dx = \int_0^1 (1-x)\, y \Big|_0^{1-x^2} dx$$

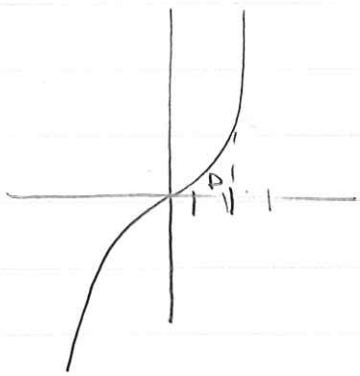
$$= \int_0^1 (1-x)\,[1-x^2]\, dx = \int_0^1 1 - x^2 - x + x^3\, dx$$

$$= x - \frac{x^3}{3} - \frac{x^2}{2} + \frac{x^4}{4}\Big|_0^1 = 1 - \frac{1}{3} - \frac{1}{2} + \frac{1}{4} = \frac{5}{12}$$

Interchange order of integration & integrate

4) $\int_0^8 \int_{\sqrt[3]{y}}^2 e^{x^4}\,dx\,dy$

Draw Domain



Type II: $0 \le y \le x^3$

$0 \le x \le 2$

New Integral $\int_0^2 \int_0^{x^3} e^{x^4}\,dy\,dx$

$= \int_0^2 e^{x^4}\, y \Big|_0^{x^3} dx$

$= \int_0^2 e^{x^4} x^3\,dx \qquad u = x^4 \quad du = 4x^3\,dx$

$= \frac{1}{4}\int_0^{16} e^u\,du = \frac{1}{4}\left(e^{16}-1\right)$

5) (b)

Volume $= V_{sphere} - V_{cylinder}$

$= \frac{4}{3}\pi r_2^3 - V_{cylinder}$

$V_{cylinder} = 2\int_0^{2\pi}\int_0^{r_1} \sqrt{r_2^2 - r^2}\; r\,dr\,d\theta$

$u = r_2^2 - r^2 \quad du = -2r\,dr$

$D =$ (r_1)

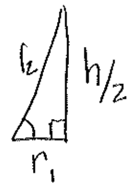
$= -\frac{2}{2}\int_0^{2\pi}\int_{r_2^2}^{r_2^2-r_1^2} \sqrt{u}\,du\,d\theta$

$= -\int_0^{2\pi} \frac{2}{3} u^{3/2}\Big|_{r_2^2}^{r_2^2-r_1^2} d\theta = -\frac{2}{3}\,2\pi\left[(r_2^2-r_1^2)^{3/2} - r_2^3\right]$

$= \frac{4}{3}\pi\left(r_2^3 - (r_2^2-r_1^2)^{3/2}\right)$

What is the height of the Ring

Inside the cylinder, we have



$$\Rightarrow \frac{h}{2} = \sqrt{r_2^2 - r_1^2}$$

$$\Rightarrow V_{cylinder} = \frac{4\pi}{3}\left(r_2^3 - \frac{h^3}{8} \right)$$

$$\Rightarrow V = V_S - V_C = \frac{4\pi}{3}\frac{h^3}{8}$$

(a) computing over the ring.

V = 2 x Volume of top.

$$= 2 \int_0^{2\pi} \int_{r_1}^{r_2} \sqrt{r_2^2 - r^2}\, r\,dr\,d\theta \qquad u = r_2^2 - r^2 \quad du = -2r\,dr$$

$$= \frac{2}{-2}\int_0^{2\pi} \int_{r_2^2 - r_1^2}^{0} \sqrt{u}\,du\,d\theta = -1 \int_0^{2\pi} \frac{2}{3} u^{3/2} \Big|_{r_2^2 - r_1^2}^{0} d\theta$$

$$= +\frac{4\pi}{3}\left(r_2^2 - r_1^2\right)^{3/2} = \frac{4\pi}{3}\frac{h^3}{8}$$

SAME AS OTHERWAY! YEAH!!

6) D = disk centered at origin w/ radius a.
What is average distance of pts in D from origin?

DRAW D

distance from origin $= \sqrt{x^2+y^2} = r$

Total distance of pts in D from origin $= T_D$

$$T_D = \iint_D r\, dA = \int_0^{2\pi} \int_0^a r^2\, dr\, d\theta$$

$$= \frac{2\pi}{3} a^3$$

$\therefore$ Average Distance is $\dfrac{T_D}{A(D)}$ $A(D)$ = Area of D.

$A(D) = \pi a^2$

$$\Rightarrow \text{Average distance} = \frac{\frac{2\pi}{3} a^3}{a^2 \pi} = \frac{2}{3} a.$$