

CLASS 1, MATH 13, SPRING 2011, 3/28/2011

1. INTRODUCTORY COMMENTS.

Instructor information. Name: Andrew Yang, Office: Kemeny 316, Email: Andrew.C.Yang@dartmouth.edu, Office phone: 646-2960, Office hours: 3:30pm - 4:30pm MWF, or by appointment (use email or talk to me in person).

Book: *Calculus*, 6th edition, by James Stewart. Available at the bookstore. This is the same book used in Math 8 in the fall (and probably also the winter). The website has some optional textbooks listed, along with relevant comments. In particular, the book *Div, grad, curl, and all that*, might be a useful supplement for the second half of the class.

Webpage: www.math.dartmouth.edu/~m13s11. This is very important as it will have a link to Webwork as well as a your weekly written homework assignments. It has comprehensive information about the class and will also be the place to go for updates as the class proceeds.

Grading: Based on homework and exams. Homework will constitute 20% of the grade, while exams the remaining 80%. There will be two midterms, each two hours long. The final will be three hours in length, with the exact date to be determined. Of the 80% of your grade determined by exams, the breakdown is 25/25/30.

There are two types of homework: Webwork and written assignments. Webwork is an entirely computer-based homework system, where you login (go to the math 13 webpage for a link to the login page) using your ID and password, and get homework problems from the Webwork system. You input your answers back into the computer, and the system will tell you if you are right or wrong. If you are wrong, you have the chance (infinitely many chances, most of the time) to find the correct answer. Each set of problems in the Webwork system will have a 'closing time', after which you will not be able to work on the problems for credit anymore. You will get a new Webwork assignment each class, so be sure to check the system daily, or at least every other day! Usually, Webwork assignments will be due at 10am four or five days after the corresponding content is covered in class.

You should have already received a Webwork ID and password. Once you receive the automated email to your Dartmouth email account, test to make sure that you can login properly. If you are having difficulty logging in contact me immediately and I will try to solve your problem.

Written homework assignments are more traditional and will be due once a week, on Wednesday at 12:30pm (the beginning of class). You can submit them to the homework boxes on the bottom floor of Kemeny, outside of room 008, or at the very beginning of class. Each week's homework will be listed on the course webpage. A grader will grade your assignments and they will usually be returned back in class, usually on the Monday after they have been turned in. On written assignments, we expect you to show all your work in a reasonably organized way. Correct answers without supporting justification will not be given full marks. Webwork and written homeworks will be worth an equal amount.

Tutorials: On Sunday, Tuesday, and Thursday nights, Zebediah Engberg will be running tutorial sessions at 7-9pm in 105 Kemeny, on Sunday, Tuesday, and Thursday. The tutorial sessions should start on Tuesday evening, at 3/29/2011.

X-hour: The X-hour for this section is on Thursday, at 1:00pm - 1:50pm. Keep this slot of time available – although we will not use it regularly, we might sporadically use an X-hour as a replacement class, if we need to catch up. More likely, the X-hour will be used for optional activities, such as exam reviews.

Late homework policy: In general, unexcused late homeworks will not be accepted for credit. The only general reasons we will grant extensions on homework are for illness or family emergencies. In these cases, please notify me before the assignment is due with the reason why you cannot turn in the assignment on time. If you have some other reason why you cannot finish an assignment on time, you can always email me and ask for an extension, although I cannot guarantee that you receive one. This late policy homework applies to both Webwork and written homeworks.

Assistance: In general, there is a good amount of assistance available for this class. There are the tutorial sessions, as well as office hours, and we are working on arranging for tutors to be available at the Tutor Clearinghouse. If you are having trouble in the class, do not hesitate to seek help.

2. GENERAL ADVICE

- The most important piece of advice is to keep up with the progress of the class. Mathematics may very well be the subject where progress at any one point is most dependent on understanding everything that came before it, so once you fall behind you will have difficulty understanding the material in subsequent classes. Make sure you remember the differential calculus parts of Math 8!
- The best way to test yourself for understanding of mathematical content is to solve problems by yourself. Of course, if you are having some difficulty and are not making progress on a problem, you should feel free to seek assistance from classmates, TAs, or the instructors, but it is worth trying each problem on your own for at least some amount of time. Even if you do not find a solution you may benefit from partial progress on the problem, or discover your mathematical weaknesses.
- As a matter of fact, if you are really serious about learning mathematics, you should attempt to do every problem in the textbook. I don't mean this literally, but I do mean that you should try any problem which does not seem trivially easy to you - and in particular, this means all of the more difficult problems at the end of each section and chapter. Of course, this is by no means expected or required to get a good grade.
- The above being said, for the best understanding of mathematical material, you should not only work homework assignments, but also discuss mathematics (with peers, TAs, or instructors) verbally. You will find that speaking, listening, reading, and writing mathematics are all a bit different from each other, and that maximal understanding only comes about when you engage in all four of these activities.
- Browse the section of a textbook that will be covered in class prior to actually attending the class. You shouldn't expect complete understanding, but some exposure to the terms and ideas before attending class will probably make class more enlightening and less confusing.
- If you start to have difficulty, do not hesitate to seek help. As mentioned earlier, falling behind is highly undesirable, and there is plenty of available assistance.
- Try to do a little bit of mathematics everyday. Of course, you really don't have a lot of choice since there are homework assignments every other day, but doing mathematics isn't very different from playing a sport or a musical instrument - unless you have a lot of experience, you need to practice daily to stay at your best. We don't expect you to work on math two hours a day, but something like thirty minutes to an hour per day, on average (outside of lectures, of course) isn't unreasonable.

3. AN INTRODUCTION TO MATH 13

Math 13 at Dartmouth is intended to be the sequel to Math 8. The second half of Math 8 covers differential calculus of functions of several variables, and so the entirety of Math 13 is devoted to studying integral calculus of functions of several variables. Today, we will briefly discuss the 'big-picture' of what you should expect to learn in Math 13, and how it is related to the calculus you learned in a first-year calculus class.

Recall that in single-variable calculus, there are either *definite* or *indefinite* integrals. The former is an expression which can be thought of as computing the area under the graph of a function, while the latter is simply an antiderivative; that is, a function whose derivative is equal to some other function. The relationship between these two concepts is given by the *Fundamental Theorem of Calculus*, which states that if $F'(x) = f(x)$, then

$$\int_a^b f(x) dx = F(b) - F(a).$$

This has probably become so ingrained in your mind that you no longer consciously think about the fact that, on the surface, definite and indefinite integrals aren't really similar at all. Indeed, a definite integral is defined as a limit of Riemann sums, which is a really unwieldy and abstract concept to work with.

If we want to generalize these ideas to functions of several variables, we will have our work cut out for us. Notice that we do not even have a notion of what it means for a function $F(x, y)$ to be the antiderivative of another function $f(x, y)$, or indeed, if this could even be made sense of. Notice that we have defined partial derivatives, and also the gradient of a function $f(x, y)$, but $\nabla f(x, y)$ is not a function which takes on real-values; rather, it is a vector-valued function on \mathbb{R}^2 .

The first half of this class will be devoted to learning how to define and calculate definite integrals of functions of several variables. In analogy to the single-variable case, such definite integrals should represent volumes under the graphs of functions. However, one complication arises in these integrals which makes evaluating them substantially harder than definite integrals of a single variable. When we integrate a function $f(x)$, we do so over an interval $[a, b]$. However, we will want to integrate functions $f(x, y)$ over not just rectangles, but more general regions, such as circles, or other regions usually defined by functions we are familiar with. Learning how to deal with this additional complication will be the main new concept we will have to learn in the first half of the class.

It is a fact that calculus is inspired by physics. As a result, we will look at a few questions which arise from classical physics, but which do not require any actual physics knowledge to solve. For instance, we will see how multiple integration can be used to calculate the mass, center of mass, moment of inertia, etc. of various objects with non-constant density, which was one of the first calculations calculus was invented to do.

The other parts of the first half of this class will be devoted to learning about how to use definite integrals in several different contexts. One such situation is when we want to integrate in coordinate systems more general than the familiar 'rectangular coordinates'. This has great practical applications when studying physics and engineering, since there are many instances where polar, cylindrical, or spherical coordinates are more 'natural' than rectangular coordinates. We will also briefly describe how to perform more general change of variables, although a full understanding of this requires some knowledge of linear algebra.

The second half of this class will be devoted to finding suitable analogues of the Fundamental Theorem of Calculus. To do so, we will have to learn about 'line' and 'surface' integrals, which arise naturally in physics when we want to calculate quantities such as work and flux. The class will culminate in the study of three great theorems, known as Green's Theorem, Stokes' Theorem, and the Divergence Theorem. These are all analogues of the Fundamental Theorem of Calculus in different situations, and while they all look quite different on the surface, closer inspection will reveal that they are related at a deep level.

For instance, single-variable calculus is well-suited to answer questions like the following: given a linear rod of varying density, calculate the area. Since density is just mass divided by volume (or in the 1-dimensional case, just length), this boils down to calculating a definite integral. But suppose we want to calculate the mass of a curved object with varying mass density?

Another related calculation arises naturally when we want to calculate work. Recall that work is defined as the dot product of force and displacement; ie, $W = \vec{F} \cdot \vec{d}$. This formula is all you need if force and displacement are constant; even if force is not constant, if displacement is linear then it is not too hard to see how to calculate work. But what if the object the force acts on travels in a non-linear fashion? This is a very common situation in physics; for instance, consider the elliptical orbits of planets around the sun. This will require us to define and study *line integrals*.

We will then define the generalization of line integrals to 2-dimensional objects. This will allow us to calculate quantities like surface areas of two-dimensional surfaces (in much the same way we can calculate arc length of curves using calculus), as well as quantities motivated by physics such as *flux*.

A lot of the mathematics in this class is closely related to physics, and in particular, electromagnetism. Although we do not presuppose knowledge of any physics in this class, students taking a multivariable electromagnetism class will probably have an advantage since they will be basically doing twice as much of the same mathematics as students only taking a math class or a physics class. Some of our examples will be motivated by physics, but we will not be teaching physics, and will only use these examples to provide physical intuition for the mathematical concepts we will study.