Matrices and Coordinates

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Equation of a plane

A plane in $\mathbb{R}^3$ is determined by a point in the plane $P_0(x_0, y_0, z_0)$ and a vector $\mathbf{n} = (A, B, C)$ that is normal (perpendicular) to the plane.

$$\begin{aligned}
\mathbf{n} \cdot \vec{P_0 P} &= (A, B, C) \cdot (x - x_0, y - y_0, z - z_0) \\
&= A(x - x_0) + B(y - y_0) + C(z - z_0) \\
&= 0
\end{aligned}$$

Operations on Matrices

An $m \times n$ **matrix** is an array of real numbers with m rows and n columns.

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} = (a_{ij})$$

The **sum** of two $m \times n$ matrices A and B is the $m \times n$ matrix C obtained by adding the corresponding entries in A and in B, that is $C = A + B = (a_{ij} + b_{ij})$.

Matrix Multiplication

If A is an $m \times n$ matrix and B is an $n \times p$ matrix then the **product** AB is the matrix where the ij-th entry is obtained by taking the dot product of the i-th row of A with the j-th column of B.

NOTE: In order to define the product of A and B we require that the number of columns of A be equal to the number or rows of B. Otherwise, the product is undefined.

Coordinate Systems

The **coordinates** of a point are the components of a tuple of numbers used to represent the location of the point in the plane or space.

For 2-dimensions:
- Choose an "origin" - (0,0).
- **Cartesian or rectangular coordinates**

$$(x, y)$$

x-horizontal and y-vertical direction

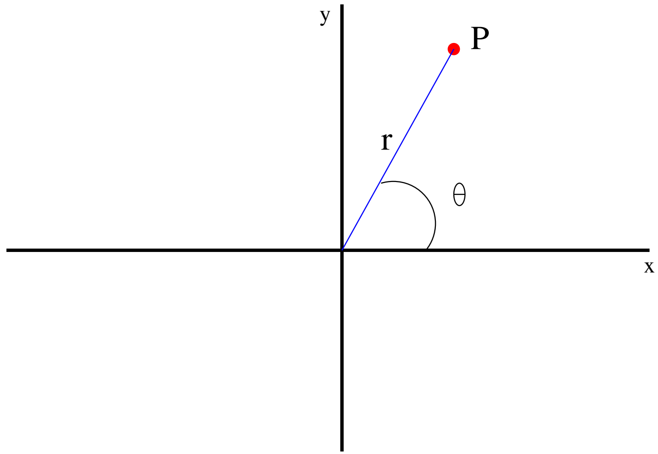
Polar coordinates: (r, θ)

r - distance from origin and
θ - angle from positive x-axis, $0 \leq \theta < 2\pi$.
If we want to describe every point uniquely we require that $r \geq 0$ and $0 \leq \theta < 2\pi$.

NOTE: In polar coordinates you think that every point except the origin is on a circle of radius r.

Polar coordinates are useful in doing computations with curves that have symmetry around the origin.

Relation between polar and cartesian coordinates



Polar to Cartesian:

$x = r\cos\theta$

$y = r\sin\theta$

Cartesian to Polar:

$r = \sqrt{x^2 + y^2}$

$\tan(\theta) = \frac{y}{x}$

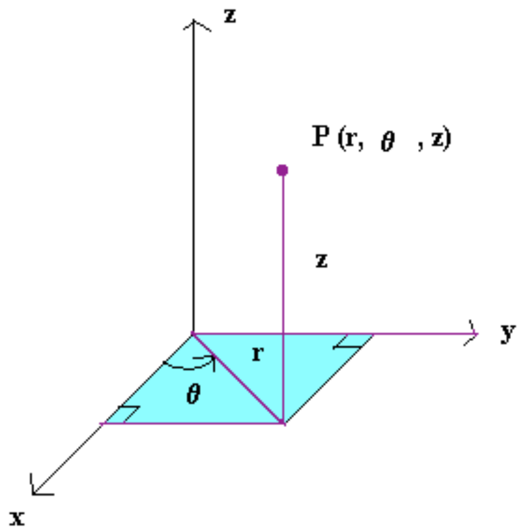
Cylindrical Coordinates: (r, θ, z)

These are for 3D. They are good for studying objects possessing an axis of symmetry.

We usually think that every point in space not on the z-axis is on a cylinder.

Cartesian to Cylindrical	**Cylindrical to Cartesian**
$x = r\cos\theta$	$r = \sqrt{x^2 + y^2}$
$y = r\sin\theta$	$\tan(\theta) = \frac{y}{x}$
$z = z$	$z = z$

Cylindrical Coordinates



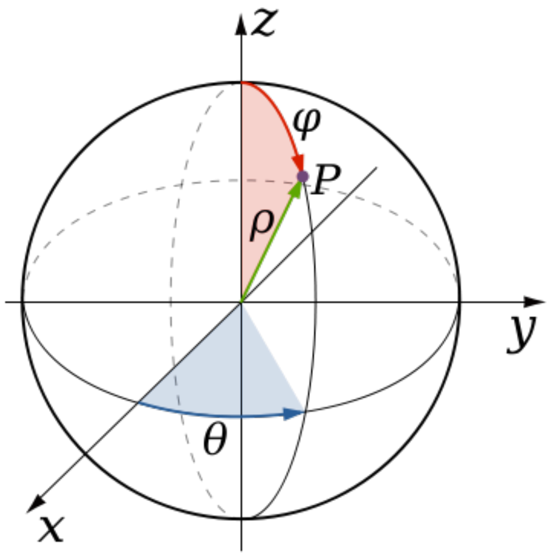
Spherical Coordinates: (ρ, ϕ, θ)

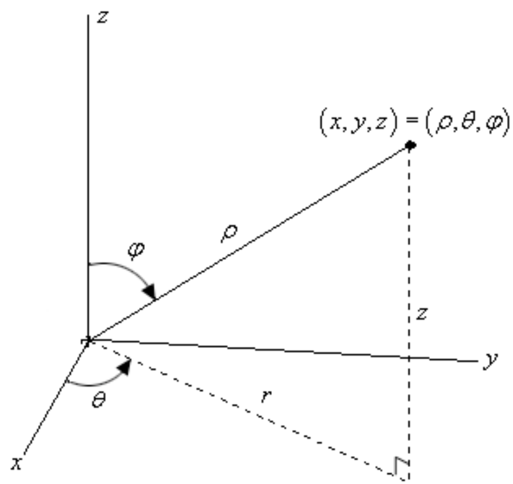
- These are to describe a point in 3D. They are useful to study objects that have a center of symmetry.

- Here we think as every point except (0,0,0) lies on a sphere.

- ρ - distance from the origin.
  ϕ - longitude and takes values $0 \leq \phi \leq \pi$.
  θ - latitude and takes values $0 \leq \theta < 2\pi$.

Spherical Coordinates





Relation between cartesian and spherical

Spherical to cartesian:

$x = \rho \sin\phi \cos\theta$

$y = \rho \sin\phi \sin\theta$

$z = \rho \cos\phi$

Cartesian to spherical:

$\rho = \sqrt{x^2 + y^2 + z^2}$

$\tan(\phi) = \sqrt{x^2 + y^2}/z$

$\tan(\theta) = \frac{y}{x}$.

Relation between cylindrical and spherical

Spherical to cylindrical:

$r = \rho \sin \phi$

$z = \rho \cos \phi$

$\theta = \theta.$

Cylindrical to spherical:

$\rho = \sqrt{r^2 + z^2}$

$\tan(\phi) = \frac{r}{z}$

$\theta = \theta.$