Surface Area

Rosa Orellana

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Standard Normal Vector

Given $\mathbf{X}(s,t) = (x(s,t), y(s,t), z(s,t))$, we have two tangent vectors:

$$\mathbf{T}_s = \frac{\partial \mathbf{T}}{\partial s} = (\frac{\partial x}{\partial s}, \frac{\partial y}{\partial s}, \frac{\partial z}{\partial s})$$

$$\mathbf{T}_t = \frac{\partial \mathbf{T}}{\partial t} = (\frac{\partial x}{\partial t}, \frac{\partial y}{\partial t}, \frac{\partial z}{\partial t})$$

Then the **standard normal vector** is

$$\boxed{\mathbf{N} = \mathbf{T}_s \times \mathbf{T}_t}$$

Smooth Surfaces

A parametrized surface S is **smooth** at a point $\mathbf{X}(s_0, t_0)$ if $\mathbf{X}$ is C^1 in a neighborhood of (s_0, t_0) and if

$$\mathbf{N}(s_0, y_0) = \mathbf{T}_s \times \mathbf{T}_t \neq \mathbf{0}$$

Surface Area

$$\text{Surface area of } S = \iint_D \|\mathbf{T}_s \times \mathbf{T}_t\| \, dsdt.$$