

- (1) For each of the following, set up (but do not evaluate) iterated integrals or quotients of iterated integral to give the indicated quantities:
- The average temperature at points in the region W lying between the paraboloids $z = 12 - x^2 - y^2$ and $z = 2(x^2 + y^2)$ if the temperature is inverse proportional to the distance from the z -axis. (Use cylindrical coordinates.)
 - The moment of inertia about the z -axis of the region lying outside the double cone $z^2 = x^2 + y^2$ and inside the ball $x^2 + y^2 + z^2 = 1$.
- (2) The base of a fence lies along the part of the parabola $y = x^2$, $1 \leq x \leq 2$, and the height of the fence at the point (x, y) is given by $h(x, y) = x$ (in feet). Find the area of the fence.
- (3) Consider the transformation $T(u, v) = (u^2 \cos(v), u^2 \sin(v))$. Describe (e.g., by a picture) how T transforms the rectangle $[0, 2] \times [0, \frac{\pi}{2}]$.
- (4) A snowball rolls downhill along a curvy path given by $\mathbf{x}(t) = (t, t^3, 25 - t^2)$, $0 \leq t \leq 5$ (where the first two coordinates denote its position east-west and north-south and the third coordinate denotes its elevation). The snowball steadily increases in size as it rolls so that its weight at time t is given by $2 + 3t$ pounds. Find the total work done by gravity on the snowball. (The gravitational force has magnitude equal to the weight of the snowball and points directly down, i.e., in the direction of $-\mathbf{k}$.)
- (5) Let D be a region bounded by a simple closed curve C in the xy -plane. Use Green's theorem to prove that the coordinates of the centroid of D are given by

$$\bar{x} = \frac{1}{2A} \int_C x^2 dy$$

and

$$\bar{y} = \frac{1}{2A} \int_C -y^2 dx$$

where A is the area of D .

- (6) Each of the following is difficult or impossible to evaluate directly but can be computed by other methods. Compute each one and justify your method.
- $\int_C 2x dx + \cos(y^2) dy$ where C is the semicircle $x^2 + y^2 = 1$, $y \geq 0$, traced from $(-1, 0)$ to $(1, 0)$.
 - $\int_C 3x^2 y^2 dx + 2x^3 y dy$ where C is the path $\mathbf{x}(t) = (e^{t^2}, t^2)$, $0 \leq t \leq 1$