

1. (10) (**Show all work**). Let  $T(x, y, z) = x^3 + y^4 - xyz^2$ . Determine whether  $T$  is increasing or decreasing at the point  $(1, -2, 1)$  in the direction of the vector  $\mathbf{u} = (1/\sqrt{3}, -1/\sqrt{3}, -1/\sqrt{3})$ .

2. (15) (**Show all work**). The level surface  $G(x, y, z) = (x - 2)^4 + (y - 2)^4 + (z - 1)^2 = 3$  and graph of  $f(x, y) = 4 - x^2 - y^2$  are two surfaces which intersect at the point  $(1, 1, 2)$ . Determine whether or not they are tangent at that point, that is, whether they have the same tangent plane.

3. (15) (Show all work).

(a) Let  $\mathbf{g}(x, y) = (x + y, x^2 - y^2, x^3y)$ . Find the derivative matrix  $D\mathbf{g}(1, 2)$ .

(b) Suppose that  $\mathbf{f} : \mathbb{R}^3 \rightarrow \mathbb{R}^2$  has derivative matrix  $D\mathbf{f} = \begin{pmatrix} 1 - vw & 1 - uw & 1 - uv \\ 2u & 2v & 1 \end{pmatrix}$ .  
With  $\mathbf{g}$  as above, find  $D(\mathbf{f} \circ \mathbf{g})(1, 2)$ .

4. (15) (**Show all work**). Consider the path  $\mathbf{c}(t) = (\sin(5t), \sqrt{3}\sin(5t), 2\cos(5t))$ .

(a) For which values of  $\alpha, \beta, \gamma$  is  $\mathbf{c}(t)$  a flowline for the vector field  $\mathbf{F}(x, y, z) = (\alpha z, \beta z, \gamma x)$ ?

(b) Compute the arclength of  $\mathbf{c}(t)$  for  $t$  from 1 to 5.

NAME (Print!): \_\_\_\_\_

Math 13

21 October 2003

First Hour Exam

Problem	Points	Score
1	10	
2	15	
3	15	
4	15	
5 (Take home)	15	
6 (Take home)	15	
7 (Take home)	15	
Total	100	