Name

MORE REVIEW QUESTIONS

First, I want to emphasize that the "name of the game" is to turn one integral into another by either filling in the curve or surface or changing the curve or surface to a different one. Stokes theorems tell you how much the original integral changes when you do this.

0.1. **Pole/Hole questions.** **1. F is defined everywhere except** $(0,0,3)$ **and** $(0,0,0)$**.** $\nabla \cdot F = 0$ **everywhere** F **is defined. If the flux through a sphere of radius 1/2 is** 12π**, and the flux through a sphere of radius 4 is** 6π**, what is the flux through a sphere of radius 5? What about a sphere of radius 10?**

2. F is defined everywhere except for the line $y = 0, z = 0$**. If** $\nabla \times F = \langle -3, 0, 0 \rangle$ **everywhere** F **is defined, and the line integral around the path** $r(t) = \langle 0, cos(t), sin(t) \rangle$ **is** 4π**, what is the the line integral of** $r(t) = \langle 0, 3cos(t), 3sin(t) \rangle$**? What about the line integral around the square whose corners are** $\langle 0, 1, -1 \rangle, \langle 0, 1, 1 \rangle$, $\langle 0, -1, 1 \rangle, \langle 0, -1, -1 \rangle$ **[say in the counter-clockwise direction]?**

0.2. Other questions. **3) In each of the following examples, find a surface or curve that has the same boundary as the curve/surface given but that would be easier to do the line integral or surface integral on. [For paths this almost always means breaking the path into 2 straight lines, or maybe a circle on which the function is nice in the components of motion, for surfaces it generally means finding a surface where the function is easy or at least easy in the components normal to the surface]. You may want to find the integral on the alternative surface. [NOTE: the curls and divergences on these questions may be icky....on the exam you would hope that the curl or divergence is nicer, but it is harder to create those, and I want you to practice finding alternative surfaces or pathes to integrate. Understand that you would still have to take the surface integral of the curl [or the volume integral of the divergence] to see how much changing the curve/surface.]**

a) $\mathbf{F}(x,y,z) = \langle (y-1)\ sin(e^{3x}\pi) + xy, (y^2-1)e^{x^2}, 0\rangle$ **on the path** $r(t) = \langle t,t,0\rangle, 0 \leq t \leq 1$**.**

b) $\mathbf{F}(x,y,z) = \langle (1-x^2-y^2-z^2), e^{1-x^2-y^2-z^2}, sin(1-x^2-y^2-z^2)\rangle$ **on the unit disk** $x^2+y^2 \leq 1, z=0$

c) $\mathbf{F}$ **is a function such that** $\nabla \times F = 0$ **everywhere except** $\langle 0,0,0\rangle$**.** $r(t) = \langle cos(t), sin(t)\rangle$**.**

d) $\mathbf{F}$ **is the paraboloid defined by** $z = 9-x^2-y^2$**,** $x^2+y^2 \leq 9$**.** $\mathbf{F}(x,y,z) = \langle zx - zy^y, ln(z^2+1), xy\rangle$**.**

e) $\mathbf{F} = \langle (x+y)z^z, e^{xy}z, z\rangle$ $r(t) = \langle sin(t), cos(t), t\rangle, 0 \leq t \leq 2\pi$**.**

4. Let $\mathbf{F} = \langle x,y,z^z\rangle$**. What is the line integral of** $\mathbf{F}$ **along the path that first goes from** $\langle 0,0,0\rangle$ **to** $\langle 0,1,0\rangle$**, then from** $\langle 0,1,0\rangle$ **to** $\langle 1,1,0\rangle$**, then from** $\langle 1,1,0\rangle$ **to** $\langle 1,0,0\rangle$**.**

5. Let $\mathbf{F} = \langle yz^2, y^2+xz^z, 3\rangle$**. What is the flux of** F **through the cylinder defined by** $x^2+y^2=4$**,** $0 \leq z \leq 10$ **PLUS its top. That is** S **is the surface made by taking the cylinder and adding its top. What is** $\iint_S \mathbf{F}\cdot d\mathbf{S}$**.**