Name

QUIZ 5

1. Let S **be the portion of the sphere** $x^2+y^2+z^2=18$ **lying above the cone** $z^2=5x^2+2y^2$**. Parameterize** S**.**

We solve for the boundary of the cone and sphere:

$$x^2+y^2+(5x^2+2y^2)=18 \rightarrow$$
$$6x^2+3y^2=18 \rightarrow$$
$$\frac{x^2}{3}+\frac{y^2}{6}=1$$

This is an equation that does not depend on y**, so we see that the region of the sphere inside the cone all lies above the elliptical disk described by the above. We thus parameterize the elliptical disk and consider the surface as a graph over that:**

Elliptical disk parametrization $G(u,v)=\langle\sqrt{3}u\cdot cos(v),\sqrt{6}u\cdot sin(v)\langle$**.**

We then simply solve for the proper z **value lying above this disk for a given** u,v**,:** $z^2=18-x^2-y^2\rightarrow$

$$z=\sqrt{18-3u^2\cdot cos^2(v)-6u^2\cdot sin^2(v)}$$

So the parametrization of the surface is:

$$G(u,vO=\langle\sqrt{3}u{\cdot}cos(v),\sqrt{6}u{\cdot}sin(v),\sqrt{18-3u^2cos^2(v)-6u^2sin^2(v)}\rangle \qquad 0\leq u\leq 1, 0\leq v\leq 2\pi$$

2. Let S **be the surface defined by** $y=x^2-z^2$ **for** $0\leq z\leq x$ **and** $0\leq x\leq 1$**. Let** $G(x,y,z)=\langle z,1,x\rangle$**. What is the vector surface integral of** G **over** S**?**

We have a surface where one equation is defined as simply a function of the other two, and those two are in a nice triangular region, so we parameterize as a graph by setting $x=u$ **and** $z=v$ **and thus getting** $y=u^2-v^2$**. Thus we have the parametrization:** $F(u,v)=\langle u,u^2{-}v^2,v\rangle$ **with limits** $0\leq u\leq 1$ **and** $0\leq v\leq u$**.**

We calculate F_u **and** F_v **[note,we have used** G_u **and** G_v **before, but now our parametrization is named** F **because I used** G **in the problem....don't be confused by this.].**

$F_u=\langle 1,2u,0\rangle$**.** $F_v=\langle 0,-2v,1\rangle$**. So** $F_u\times F_v=\langle 2u,-1,-2v\rangle$**. Then we use the standard template for a vector surface integral:**

$$\iint_S \mathbf{G}\cdot\mathbf{dS}=\int_0^1\int_0^u \mathbf{G}(\mathbf{F}(\mathbf{u},\mathbf{v}))\cdot(F_u\times F_v)dvdu=$$
$$\int_0^1\int_0^u\langle v,1,u\rangle\cdot\langle 2u,-1,2v\rangle dvdu=\int_0^1\int_0^u -1dvdu=\frac{-1}{2}$$