First technique.

- If $\mathbf{F} = \langle P(x,y,z), Q(x,y,z), R(x,y,z)\rangle = \nabla f$ then we must have

  1. $\frac{\partial f}{\partial x} = P(x,y,z)$
  2. $\frac{\partial f}{\partial y} = Q(x,y,z)$
  3. $\frac{\partial f}{\partial z} = R(x,y,z)$

- We will use the example $\mathbf{F} = \langle yz, xz + z, xy + 2z + y\rangle$

- We thus set $f = \int P(x,y,z)dx + g(y,z) = \int yzdx + g(y,z)$

- From this we get $f = xyz + g(y,z)$.

- From the second equation we then have $xz + \frac{\partial g}{\partial y} = xz + z \to \frac{\partial g}{\partial y} = z$.

- Thus we have that $g(x, y) = \int z dy + h(z)$. So $g(x, y) = yz + h(z)$.

- Since $f = xyz + g(y, z)$ we then have $f = xyz + yz + h(z)$

- From the third equation we have $xy + y + \frac{dh}{dz} = xy + 2z + y$.

- Thus we have $h(z) = \int 2z dz = z^2$.

- Putting all of this together, we have: $f = xyz + yz + z^2$.

Second Technique

- If we remember that $\int_a^b \mathbf{F} \cdot \mathbf{r}'\mathbf{ds} = f(r(b)) - f(r(a))$, and we allow ourselves the luxury of setting $f(\langle 0,0,0\rangle) = 0$, then we see that we can calculate $f(\langle a,b,c\rangle)$ quickly by simply picking a path from $\langle 0,0,0\rangle$ to $\langle a,b,c\rangle$ that is particularly easy to use.

- . In particular we will use a path that first goes from $\langle 0,0,0\rangle$ to $\langle a,0,0\rangle$, then a path from $\langle a,0,0\rangle$ to $\langle a,b,0\rangle$, and then finally from $\langle a,b,0\rangle$ to $\langle a,b,c\rangle$.

- We will add the integrals over these three trips up to get the integral of the full path [and hence the value of f.

- We use the same example as earlier: $\mathbf{F} = \langle yz, xz+z, xy+y+2z\rangle$.

- For the first leg $r(t) = \langle t, 0, 0\rangle$ $0 \leq t \leq a$. Thus $\int_0^a = \mathbf{F} \cdot r'(t)dt = \langle (0)(0), t(0) + 0, t(0) + 0 + 2(0)\rangle \cdot \langle 1, 0, 0\rangle = \int_0^a (0)(0)dt = 0$ [note that y=0,z=0 on this path...that is where the 0's came from].

- For the second leg $r(t) = \langle a, t, 0\rangle$ $0 \leq t \leq b$. Thus $\int_0^b \mathbf{F} \cdot \mathbf{r}'(\mathbf{t})dt = \int_0^b \langle t(0), a(0) + 0, at + t + 2(0)\rangle \langle 0, 1, 0\rangle = \int_0^b 0dy = 0$ [Note on this leg $x = a$ and $z = 0$, which is where those letters came from].

- For the third leg $r(t) = \langle a, b, t\rangle$ for $0 \leq t \leq c$, so $\int_0^c \mathbf{F} \cdot \mathbf{r}'(\mathbf{t})dt = \int_0^c ab + b + 2tdt = abc + bc + b^2 - ab(0) - b(0) - 0^2 = abc + bc + c^2$.

- Since the point $\langle a, b, c\rangle$ was any point, and $f(\langle a, b, c\rangle)$ must equal the value of this integral, we get that $f(\langle x, y, z\rangle = xyz + yz + z^2$.