## Arc Length

- If one thinks of a curve as the path of a car driving along, the arc length is what the odometer reads.
- We generally use $s$ to stand for arc length
- $\left|\mathbf{r}^{\prime}(t)\right|=\frac{d s}{d t}$.
- $s(T)=\int_{0}^{T}\left|\mathbf{r}^{\prime}(t)\right| d t$.
- We often wish to reparametrize a curve in terms of $s$, in which case we use the following recipe:
- find $\left|\mathbf{r}^{\prime}(t)\right|$
- Calculate $s(T)$ for an arbitrary choice of T
- Massage the answer to get $T$ in terms of $s$ [rather than $s$ in terms of $T$.
- Since $T$ can be any number non-negative value the above actually gives a formula for $t$ in terms of $s$.
- substitute this formula into the formula for $\mathbf{r}(t)$ to get a formula $\mathbf{r}(s)$ for $\mathbf{r}$ in terms of $s$.

