Generalized Derivative

- Is a generalization of the "normal" derivative in the sense that it approximates the change in the range given a change in the domain.

- If there is any hope at all, the derivative of a function is the derivative matrix, whose entries are the partial derivatives. <u>rows</u> correspond to <u>range</u> variables and <u>columns</u> correspond to <u>domain</u> variables.

- The above matrix may not work because it is possible that simply knowing the change in each of the "co-ordinate" directions is not enough to describe the local behavior of the function.

- If $\mathbf{F} : \Re^m \to \Re^n$ is a function. We say that $\mathbf{F}$ is differentiable at a point $\widehat{q} = \langle q_1, q_2, \ldots, q_m \rangle$ if $\lim_{\widehat{h} \to \hat{0}} \mathbf{F}(\hat{\mathbf{q}} + \hat{\mathbf{h}}) - \mathbf{F}(\hat{\mathbf{q}}) - D\mathbf{F}_{\widehat{q}} \cdot_{matrix} \widehat{h}$ goes to $\hat{0}$ "faster" than $\widehat{h}$ does.

- It is sufficient to find that the each component of the difference contains more than 1 power of a component of h in it. So if the difference is $\langle h_1 h_2, 2h_2^2, 0 \rangle$, then that is good enough.

- The derivative approximates the change in the function in the sense that $DF \cdot_{matrix} \widehat{a}$ represents the approximate change in the function as you move $\widehat{a}$ away from the base point.

- With the above in mind we can say that the velocity in the range [which is a vector] of the image of a path $\mathbf{r(t)}$ in the domain can be found by computing $DF \cdot_{matrix} \mathbf{r'(t)}$.