Poles and Holes

- if a function is not defined at a point, we say it has a ”hole” there.

- Of particular use to us is when it ”blows up” at that point [that is to say the function gets bigger and bigger without bound, almost always due to a function that goes to zero in the denominator.] These points are called ”poles.”

- If a function has a hole somewhere inside a region, the standard tricks can’t always be used because you can’t ”fill in” a region that ”isn’t there.”

- In this case we can still say:

$$\iint_{\partial V} \mathbf{F} \cdot \mathbf{dS} = \iiint_V \nabla \cdot \mathbf{F} dV + \text{contribution due to holes}$$

A similar modification of Stokes' theorem is also true.

- Of particular use is when the divergence [or curl] of a function is 0, as that means two surfaces [or loops] will have the same integral [though not necessarily 0] if they have aligned orientations and enclose the same holes.

- Even when the divergence [or curl] is not zero, we can still get some use out of this if it is easy to calculate the flux/circulation due to the divergence [or curl] [because then we can figure out how much of the flux [or circulation] is due to the holes by subtraction].