Boundary Operator

- The boundary operator "∂" is a natural way of taking solid to a surface, a surface to a collection of curves, or a collection of curves to a collection of [signed] points.

- The boundary of a boundary is [for our purposes] always the empty set.

- Boundaries are oriented in a way so that curves satisfy the right-hand rule...that is the curve goes counter-clockwise if one is looking from the direction opposite the orientation of the surface. When taking the boundary of a line, endpoints are "positive" and initial points are "negative."

- if we consider the differential operator ∇ as a "vector" equal to $\langle\frac{\delta}{\delta x},\frac{\delta}{\delta y},\frac{\delta}{\delta z}\rangle$ where we "multiply" a $\frac{\delta}{\delta x}$ [for example] by taking the derivative with respect to x, then we have a natural way of taking scalar FUNCTIONS to vector FUNCTIONS : $f \rightarrow \nabla f$. Similarly we have a way of taking vector FUNCTIONS to vector FUNCTIONS: $F \rightarrow \nabla \times F$. Finally we have a method of taking vector FUNCTIONS to scalar FUNCTIONS: $F \rightarrow \nabla \cdot F$.

- Doing this operation two times in a row always gives the 0 function.

Stokes Theorem

- Stokes' theorems are generalizations of the fundamental theorem of calculus.

- Stokes' Theorems say that you may "fill in" the curve, surface, points, etc you are integrating upon if you apply the Nabla operator to the function. Here to "integrate" on a point means to simply evaluate the function at the point. In other words:
  - If $F = \nabla f$ then $\int_a^b F(r(t)) \cdot dt = f(r(b)) - f(r(a))$ —Fundamental Theorem of Line Integrals
  - If $F = \nabla \times G$ then $\iint_S F \cdot dS = \int_{\partial S} G \cdot dr$.
  - If $f = \nabla \cdot G$ then $\iiint_V g dV = \iint_{\partial V} F \cdot dS$.