Vector Surface Integrals

- if $G(u,v) \to \langle g_1, g_2, g_3 \rangle, a \leq v \leq b, c(v) \leq u \leq d(v)$ is a parametrization of a surface S, and F is some vector valued function, then the surface integral of F on the surface S is

$$\int\int_S F \cdot dA = \int_a^b \int_{c(v)}^{d(v)} F(G(u,v)) \cdot (G_u \times G_v)\, du dv$$

- If the parametrization is with $a \leq u \leq b, c(u) \leq v \leq d(u)$, then the surface integral is:

$$\int_a^b \int_{c(u)}^{d(u)} F(G(u,v)) \cdot (G_u \times G_v)\, dv du$$

- The value of the surface integral is the flow or flux through the surface.

- The value of $G_u \times G_v$ is the $orientation$ of the surface...if it is opposite that of the flow, the integral will be negative.