Surface Integral

- if $G(u,v) = \langle g_1(u,v), g_2(u,v), g_3(u,v)\rangle$ is a parametrization, then $G_u = \langle \frac{\partial g_1}{\partial u}, \frac{\partial g_2}{\partial u}, \frac{\partial g_3}{\partial u}\rangle$ and $G_v = \langle \frac{\partial g_1}{\partial v}, \frac{\partial g_2}{\partial v}, \frac{\partial g_3}{\partial v}\rangle$ represent tangent lines on the surface that correspond to movement in the u and v directions in the range. In particular $G_u \times G_v$ is a vector whose magnitude represents the amount that the (u,v) plane is being stretched out to make the surface. It is the 2 dimensional version of velocity [kind of].

- If S is a surface parametrized by $G(u,v) = \langle g_1(u,v), g_2(u,v), g_3(u,v)\rangle$ where u,v span a region R, then the surface integral of a function $f(x,y,z)$ on the surface S can be calculated by:

$$\int\int_S f dA = \int\int_R f(G(u,v)) \mid G_u \times G_v \mid dudv$$

- The above gives a method to calculate the surface are of any surface by simply taking $f = 1$.

- The above also gives a way to easily figure out the proper area differential product for arbitrary change of variables for 2 dimensional surfaces, as you can consider any such change of variables as a parametrization of a surface. For example the change of variables from polar to cartesian would be $G(u, v) = \langle u \cdot cos(v), u \cdot sin(v), 0\rangle$ [Note: the $\cdot$ is regular multiplication]. Here u would be the variable whose name is normally r and v is the variable whose name is normally θ. And if you calculate it you will find that $\mid G_u \times G_v \mid = u$, showing that the "finagling factor" for polar is the radius.

Alternate version for 2 variable to 2 variable transformations.

- Consider the change of variables $G(u, v) = \langle g_1(u, v), g_2(u, v)\rangle$, make the matrix $\begin{bmatrix} \frac{\partial g_1}{\partial u} & \frac{\partial g_1}{\partial v} \\ \frac{\partial g_2}{\partial u} & \frac{\partial g_2}{\partial v} \end{bmatrix}$

- This matrix encodes the local geometry of the transformation in a way we will study soon. The determinant of the matrix represents the stretching of the transformation.

- Note, in class I may have switched the upper right-hand and lower-left-hand entries....either way will give the correct answer, but the one I have here is better as a lead-in to what we will be doing later.