

## Double Integrals [continued]

- To use the double integral on a surface in the plane, one must break the region into regions that are "type 1" or "Type 2."
- A type 1 region is one there are never more than 2 boundary points associated with any particular  $x$  value. That is to say for any vertical line drawn, the line will never intersect the boundary in more than 2 places.
- A type II region is similar, except now with  $y$  instead. So any horizontal line will cross the boundary at no more than 2 points.
- If a region is Type I and not Type II, you must integrate in an order such that  $dx$  is the final differential.

- If a region is Type II and not Type I, you must integrate in an order such that  $dy$  is the final differential.
- If a region is both Type I and Type II, either order will work.

Average of functions.

- Similar to the case for regular functions and functions evaluated along curves, the average value of a function  $f$  over a region  $R$  is

$$\frac{\int \int_R f \, dA}{\int \int_R 1 \, dA}$$

- If you have trouble picturing how the average works, always think of the  $f$  as a density function...and then the upper integral

is total mass and the lower integral is total area [in this case]. It should then make sense that the average density is this ratio

## Probability Density

- Probability Density is a measure of how likely an event is in the vicinity of a given point. In particular think of probability density as very similar to regular density. Regular density tells you how massive an object is per [length/area/volume] at the point under consideration. Probability density gives a measure of likeliness. If your density is  $2 \text{ kg}/\text{m}^2$  it is saying "If the wire were this dense for 2 1 square meter of surface then that 1 square meter portion of the surface would have massive equal to 2 kg." Similarly, a Probability Density of  $.4/\text{m}^2$  says

that, if the event is equally likely in a "region" of 1 square meter, then there is a .4 [40 percent] chance the event will happen in that region.

- The integral of Probability Density over the entire region under review is always 1. [Note if you restrict your purview to a smaller region, the probability may be less.]