Chapter 16, section 9, number 18:

We are told to use the parametrization $G(u,v,w) = \langle au, bv, cw\rangle, 0 \le u \le 1, 0 \le v \le 1, 0 \le w \le 1$. The Jacobian for this transformation is $\begin{vmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{vmatrix} = abc.$

We must decide what the proper limits of integration are. We choose the order $dw\ dv\ du$. Since we have that $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, plugging in the parametrization variables give $u^2 + v^2 + w^2 = 1$. The absolute bounds on u are -1 and 1 For a fixed u we have that the biggest and smallest possible values for v occur when $w = 0$ and in that case we have that $v = -\sqrt{1-u^2}$ and $v = \sqrt{1-u^2}$. Finally for a fixed u and v, w can range from $-\sqrt{1-u^2-v^2}$ to $\sqrt{1-u^2-v^2}$. Hence the triple integral we are looking for is:

$$\int_{-1}^{1}\int_{-\sqrt{1-u^2}}^{\sqrt{1-u^2}}\int_{-\sqrt{1-u^2-v^2}}^{\sqrt{1-u^2-v^2}} (au)^2(bv)(abc\ dw\ dv\ du)$$

Since there is no w in the integrand this is:

$$\int_{-1}^{1}\int_{-\sqrt{1-u^2}}^{\sqrt{1-u^2}} 2\sqrt{1-u^2-v^2}(au)^2(bv)(abc\ dv\ du)$$

By using a substitution: $m = 1 - u^2 - v^2$ we get that this equals

$$\int_{-1}^{1} -a^2u^2babc\frac{2}{3}\left(\sqrt{1-u^2-v^2}\right)^{3/2}\Big|_{-\sqrt{1-u^2}}^{\sqrt{1-u^2}} dv\ du = 0$$

17.8 Problem 3.

We are asked to evaluate $\iint_S \nabla\mathbf{F} \times \mathbf{dS}$ For S the upper hemisphere oriented upwards. From Stokes theorem we have $\iint_S \nabla\mathbf{F} \times \mathbf{dS} = \int_{\partial S} \mathbf{F} \cdot \mathbf{dr}$. ∂S is the circle of radius 2 in the xy plane, centered at the origin. Since S is oriented with upward pointing normal vector, we must have that ∂S is the circle traversed in the counter-clockwise direction, so $\mathbf{r(t)} = \langle 2cos(t), 2sin(t), 0\rangle$ suffices as a parametrization. Since when $z = 0$ we have that $\mathbf{F} = \langle x^2, y^2, 0\rangle$, we then have that

$$\int_{\partial S} F\cdot dr = \int_0^{2\pi} \langle 4cos^2(t), 4sin^2(t), 0\rangle \cdot \mathbf{r'(t)}dt = \int_0^{2\pi} \langle 4cos^2(t), 4sin^2(t), 0\rangle \cdot \langle -sin(t), cos(t), 0\rangle dt =$$

$$\int_0^{2\pi} -4cos^2(t)sin(t) + 4sin^2(t)cos(t)dt = \frac{4}{3}\left(cos^3(t) + sin^3(t)\right)\Big|_0^{2\pi} = 0$$

17.9 problem 7.

We are asked to calculate the flux through cube of side length 2 centered at the origin of the function $\mathbf{F} = \langle 3y^2z^3, 9x^2yz^2, -4xy^2\rangle$. $\nabla \cdot \mathbf{F} = 0 + 9x^2y^2 + 0$ and the divergence theorem says $\iint_{\partial V} \mathbf{F} \cdot \mathbf{dS} = \iiint_V \nabla \cdot \mathbf{F} dV$.

Since we have a cube we can use the parametrization $G(u,v,w) = \langle x, y, z\rangle$. Since the Jacobian in this very special case is 1 we have simply.

$$Flux = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} 9u^2w^2 du\ dv\ dw = 8$$