HW 18

1. Show that the following functions are differentiable at the point $\langle 2, 1\rangle$.
a) $F(u, v) = \langle u, 3u^2 v, v\sqrt{u}\rangle$

b) $F(u, v) = \langle 2u, 6u + 2v + 3, u - 3v - 2, v\rangle$ [Note: This is a "linear" function [just constants and scalar multiples], this is the equivalent of a straight line. Thus, the approx. by derivative matrix <u>should</u> be perfect.]

2. At the end of the class I attempted to give some motivation for what the velocity in the range should be the multiple of the derivative matrix times the velocity in the domain. This would correspond to the idea that if the slope is 4 and you are going 2 m/s in the x direction, then you should be going $4 \cdot 2$ m/s in the y direction. Here we wish to observe that direction.

a) Let $r(t) = (3t, 2t^2)$ then at $t = 2$, $r(t) = \langle 6, 8\rangle$. We think of $r(t)$ as moving around in the u - v plane. Let $F(u, v) = \langle uv^2, v - u\rangle$ be function from $\Re^2 \to \Re^2$. What is the "image" of $r(t)$ at $t = 2$? That is to say what is $\mathbf{F(r(2))}$? What is the image of $r(t)$ at $t = 2.01$? These two answers should be vectors in the <u>*range*</u>. From this what is the approx. velocity of the image? [To do this you should subtract the image at of $r(2)$ from the image of $r(2.01)$ and then divide by .01].

Now, use the idea that velocity of image $= DF_{\langle 6,8\rangle} \cdot_{\text{matrix}} r'(t)$ where we think of $r'(t)$ as a column vector. You should get similar answers.

b) Do the same for $r(t) = \langle sin(t), cos(t)\rangle$ and $\mathbf{F}(u, v) = \langle u^2 + v^2, u/v, v^2 - u^2\rangle$ at $t = \pi/4$.