NAME : Key

Math 13

Final Exam
November 20, 2016

Prof. Pantone

INSTRUCTIONS: This is a closed book exam and no notes are allowed. You are not to provide or receive help from any outside source during the exam except that you may ask the instructor for clarification of a problem. You have 180 minutes and you should attempt all problems.

- Print your name in the space provided.
- Calculators or other computing devices are not allowed.
- Except when indicated, you must show all work and give justification for your answer. **A correct answer with incorrect work will be considered wrong.**

All work on this exam should be completed in accordance with the Dartmouth Academic Honor Principle.

Helpful Hints:

- Use your time wisely. If you're worried about having enough time, start with the problems that are worth more points.
- Work carefully and methodically to prevent arithmetic errors.
- Work neatly so you have enough room.
- Don't Panic!

Problem	Points	Score
1	6	
2	32	
3	7	
4	7	
5	6	
6	6	
7	6	
8	0	
9	0	
Total	70	

Bonus,
3pts each

Section 1: True/False.

1. (6) Choose the correct answer. *No justification is required for your answers. No partial credit will be awarded.*

(a) The correction factor for $\underbrace{\text{polar}}_{r}$ integrals is the same as the correction factor for $\underbrace{\text{spherical}}_{\rho^2\sin(\varphi)}$ integrals.

True $\qquad\qquad$ $\boxed{\text{False}}$

(b) When computing the integral of a scalar function over a surface $\left(\iint_S f(x,y,z)\,dS\right)$, the surface S must have an orientation.

True $\qquad\qquad$ $\boxed{\text{False}}$

We only need $\|\vec{N}\|$, not $\vec{N}$.

(c) The vectors $\langle 3, 6, -1\rangle$ and $\langle 2, 2, 18\rangle$ are orthogonal.

$\boxed{\text{True}}$ $\qquad\qquad$ False

$\langle 3,6,-1\rangle \cdot \langle 2,2,18\rangle = 6+12-18 = 0$ ✓

(d) For all vector fields $\boldsymbol{F}$, $\text{curl}(\underbrace{\text{div}(\boldsymbol{F})}_{\text{scalar function}}) = \mathbf{0}$.

True $\qquad\qquad$ $\boxed{\text{False}}$

You can't take the curl of a scalar function.

(e) Stokes' Theorem cannot be applied to surfaces S that are completely enclosed (i.e., no boundary).

True $\qquad\qquad$ $\boxed{\text{False}}$

It can, and you get zero.

(f) The Divergence Theorem can only be applied to surfaces S that are completely enclosed (i.e., no boundary).

$\boxed{\text{True}}$ $\qquad\qquad$ False

Part of the hypothesis.

Section 2: Multiple Choice.

2. (32) *Circle your answer. No justification is required. No partial credit will be awarded.* **If it is not obvious which answer you have circled, you won't receive credit.**

(a) Let W be the three-dimensional solid that is a pyramid with a square base with corner points $(0,0,0)$, $(2,0,0)$, $(0,2,0)$, and $(2,2,0)$ and peak $(1,1,5)$. Let $\boldsymbol{F} = \langle x^2(y - e^z), -xy^2, 2xe^z \rangle$. Let S be the surface of W with outward-facing normal vectors. Calculate $\iint_S \boldsymbol{F} dS$.

(I) -1

(II) $-1/5$

$\boxed{\text{(III)} \quad 0}$

(IV) $1/5$

(V) 1

$\operatorname{div}(\vec{F}) = 2xy - 2xe^z - 2xy + 2xe^z$

$= 0.$

By DT, $\iint_S \vec{F} dS = \iiint_W 0 \, dV = 0.$

(b) Let $\boldsymbol{F} = \langle e^x \sin(y), e^x \cos(y) + x \rangle$. Let C be the boundary of the region between the graphs of $y = x^2$ and $y = 1$, oriented counterclockwise. Find $\int_C \boldsymbol{F} d\boldsymbol{r}$.

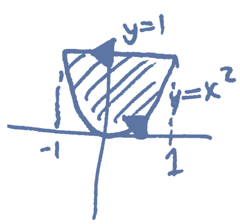
(I) 0

(II) 1

$\boxed{\text{(III)} \quad 4/3}$

(IV) $5/3$

(V) 2



By Green's Theorem:

$\oint_C \vec{F} d\vec{r} = \iint_D \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right) dA$

interior of C

$= \int_{-1}^{1} \int_{x^2}^{1} (e^x \cos(y) + 1 - e^x \cos(y)) \, dy \, dx$

$= \int_{-1}^{1} \int_{x^2}^{1} 1 \, dy \, dx = \int_{-1}^{1} (1 - x^2) dx$

$= \left[x - \frac{x^3}{3} \right]_{-1}^{1} = \frac{4}{3}$

(c) Let D be a region in the xy-plane with area equal to 7. Compute the surface area of the part of the plane $3x - y + 2z = 10$ lying over the region D.

(I) 7

(II) $\dfrac{14}{\sqrt{14}}$

(III) $\dfrac{7}{2}$

(IV) $\dfrac{\sqrt{14}}{2}$

(V) $\boxed{\dfrac{7\sqrt{14}}{2}}$

Area (D) = 7.

Surface area $= \iint_S 1\,dS = \iint_D \|\vec{N}\|\,dA$.

plane: $z = 5 - \frac{3}{2}x + \frac{y}{2}$

param: $\langle x, y, 5 - \frac{3}{2}x + \frac{y}{2} \rangle$

$T_x = \langle 1, 0, -\frac{3}{2} \rangle$

$T_y = \langle 0, 1, \frac{1}{2} \rangle$

$\vec{N} = \langle \frac{3}{2}, -\frac{1}{2}, 1 \rangle$.

$\|\vec{N}\| = \sqrt{\frac{9}{4} + \frac{1}{4} + \frac{4}{4}} = \frac{\sqrt{14}}{2}$.

$= \frac{\sqrt{14}}{2} \iint_D 1\,dA$

$= \frac{\sqrt{14}}{2} \cdot \text{Area}(D)$

$= \frac{7\sqrt{14}}{2}$

(d) The point $(2, 2, 5)$ in Cartesian coordinates is equal to $(r, \theta, z) =$ __________ in cylindrical coordinates.

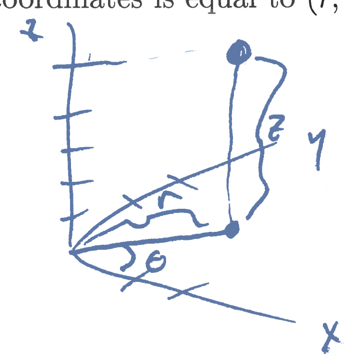
(I) $(\pi/8, \pi/4, \pi/6)$

(II) $\boxed{(\sqrt{8}, \pi/4, 5)}$

(III) $(2, \pi/2, 5)$

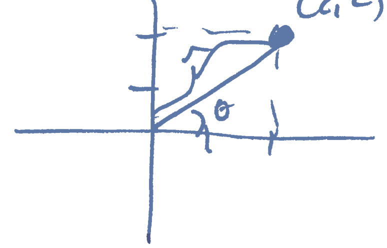
(IV) $(\sqrt{8}, \pi/2, 5)$

(V) $(\sqrt{2}, \pi/2, 5)$



$z = 5$.

r, θ:



$r = \sqrt{x^2 + y^2}$

$= \sqrt{4+4} = \sqrt{8}$.

$\theta = \arctan\left(\frac{2}{2}\right)$

$= \pi/4$

(e) Calculate $\displaystyle\iint_R \frac{\cos(x)}{y}\,dA$ for $R = \left[0, \frac{\pi}{4}\right] \times [1, e^2]$.

(I) $\ln(2)$

(II) $\sqrt{2}$

(III) 2

(IV) 1

(V) $\ln(2) - 1$

$$= \left(\int_0^{\pi/4} \cos(x)\,dx\right)\left(\int_1^{e^2} \frac{1}{y}\,dy\right)$$

$$= \Big[\sin(x)\Big]_0^{\pi/4} \cdot \Big[\ln(y)\Big]_1^{e^2}$$

$$= \left(\frac{\sqrt{2}}{2} - 0\right)\left(\underbrace{\ln(e^2)}_{2} - \underbrace{\ln(1)}_{0}\right)$$

$$= \frac{\sqrt{2}}{2} \cdot 2 = \sqrt{2}$$

(f) Let $\boldsymbol{F} = \langle yz + y^2ze^{xz},\ xz + 2ye^{xz},\ xy + xy^2e^{\overset{xz}{\cancel{xy}}}\rangle$. Find a potential function for $\boldsymbol{F}$, if one exists.

(I) $f(x, y, z) = xyz + y^2e^{x+z}$

(II) $f(x, y, z) = xyz + y^2e^{xyz}$

(III) $f(x, y, z) = xyz + y^2e^{xz}$

(IV) $f(x, y, z) = xyz + xyze^{xyz}$

(V) $\boldsymbol{F}$ does not have a potential function.

If $f = xyz$ then $\vec{F} = \langle yz, xz, xy\rangle$, then that part matches.

What's left is $\langle y^2ze^{xz}, 2ye^{xz}, xy^2e^{xz}\rangle$,

and this is $\nabla(y^2e^{xz})$.

(g) Which of these integrals computes the volume of a sphere of radius R?

(I) $\int_0^{2\pi}\int_0^{\pi/2}\int_0^R \rho^2\sin(\phi)\,d\rho d\phi d\theta$ } half-sphere

(II) $\int_0^{2\pi}\int_0^{\pi/2}\int_0^R 1\,d\rho d\phi d\theta$ } no correction factor

(III) $8\int_{3\pi/2}^{2\pi}\int_0^{\pi/2}\int_0^R \rho^2\cos(\phi)\,d\rho d\phi d\theta$ } wrong correction factor

$\boxed{\text{(IV)}\ 8\int_{3\pi/2}^{2\pi}\int_0^{\pi/2}\int_0^R \rho^2\sin(\phi)\,d\rho d\phi d\theta}$ } $= 8\cdot$(vol of $\frac{1}{8}$ of sphere).

(V) $\int_0^{2\pi}\int_0^{\pi}\int_0^R 1\,d\rho d\phi d\theta$ } no correction factor.

(h) Use the Divergence Theorem to calculate the surface integral $\iint_S \langle x^3, y^3, z^3\rangle dS$ where S is the surface of the sphere $x^2+y^2+z^2=1$ with outward-facing normal vectors.

(I) 0

$\boxed{\text{(II)}\ \frac{12\pi}{5}}$

(III) $-\frac{6\pi}{5}$

(IV) $\frac{6\pi}{5}$

(V) None of the above

Let W = solid sphere.

By DT: (outward, ✓)

$$\iint_S \vec{F}\,d\vec{S} = \iiint_W \operatorname{div}(\vec{F})\,dV$$

$$= \iiint_W 3(x^2+y^2+z^2)\,dV$$

$$\xrightarrow{\text{spherical}} = 3\int_0^{2\pi}\int_0^{\pi}\int_0^{1} \rho^4\sin(\varphi)\,d\rho\,d\varphi\,d\theta$$

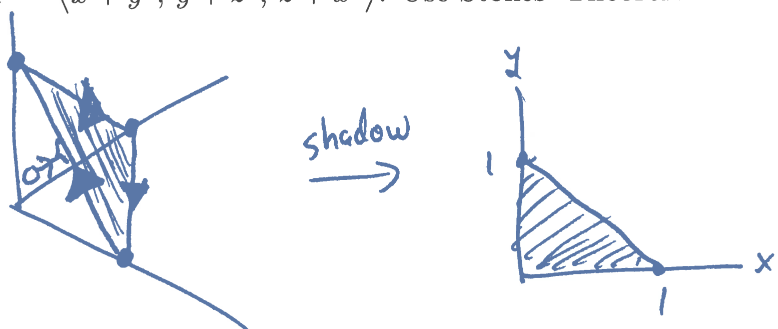
$$= 3\left(\int_0^{2\pi} d\theta\right)\left(\int_0^{\pi}\sin(\varphi)\,d\varphi\right)\left(\int_0^1 \rho^4\,d\rho\right) = 3\cdot 2\pi\cdot\left[-\cos(\varphi)\right]_0^{\pi}\cdot\left[\frac{\rho^5}{5}\right]_0^1$$

$$= 3\cdot 2\pi\cdot 2\cdot\frac{1}{5} = \frac{12\pi}{5}$$

Section 3: Long Answer.

You must show all work to receive credit. If you need more space you may use the back of the page. You must clearly indicate on the front of the page that there is more work on the back of the page. Please work neatly and circle your final answer.

3. (7) Let C be the triangle with corner points $(1,0,0)$, $(0,1,0)$, and $(0,0,1)$, oriented clockwise when viewed from above. Let $\boldsymbol{F} = \langle x + y^2,\, y + z^2,\, z + x^2 \rangle$. Use Stokes' Theorem to calculate $\oint_C \boldsymbol{F}\, d\boldsymbol{r}$.



By Stokes' Theorem: $\oint_C \vec{F} = \iint_S \mathrm{curl}(\vec{F})\,dS$, where S is the interior of the triangle with downward-pointing normal vectors.

Eq for the triangle:

$$\begin{array}{c} \vec{i} \;\; \vec{j} \;\; \vec{k} \\ \vec{v}_1 = \langle -1,1,0\rangle \\ \vec{v}_2 = \langle 0,-1,1\rangle \\ \vec{v}_1 \times \vec{v}_2 = \langle 1,1,1\rangle. \end{array}$$

So, $x+y+z=k$

plug in $(1,0,0)$

$1+0+0=k$

$k=1$.

$\Downarrow$

$x+y+z=1 \Rightarrow z = 1-x-y$

param $\langle x, y, 1-x-y\rangle$

$T_x = \langle 1,0,-1\rangle$

$T_y = \langle 0,1,-1\rangle$

$\vec{N} = \langle 1,1,1\rangle$. Need downward! $\langle -1,-1,-1\rangle$

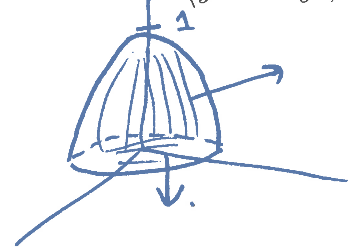
$$\mathrm{curl}(\vec{F}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x+y^2 & y+z^2 & z+x^2 \end{vmatrix} = \langle -2z, -2x, -2y\rangle.$$

$$\iint_S \mathrm{curl}(F)\,dS = \int_0^1\int_0^{1-x} \langle -2(1-x-y), -2x, -2y\rangle \cdot \langle -1,-1,-1\rangle\,dy\,dx = \int_0^1\int_0^{1-x} 2\,dy\,dx$$

$$= \int_0^1\int_0^{1-x} (2 \cancel{-2x} \cancel{-2y} + \cancel{2x} + \cancel{2y})\,dy\,dx$$

$$= 2\int_0^1 (1-x)\,dx = \boxed{1}$$

4. (7) Let W be the solid region bounded by $z = 0$ and the paraboloid $z = 1 - x^2 - y^2$. Let S be the boundary of W with outward-facing normal vectors. Let $\boldsymbol{F} = \langle y^2 z + xy^2, xe^z + x^2 y, ye^{yx} \rangle$. Compute $\iint_S \boldsymbol{F}\, d\cancel{r}\,S$.



S = outside of paraboloid and bottom disk.

Direct application of Divergence Theorem. (outward, ✓)

$\operatorname{div}(\vec{F}) = y^2 + x^2.$

$$\iint_S \vec{F}\, dS = \iiint_W (x^2+y^2)\, dV.$$

Cylindrical coordinates: $z = 1 - r^2\cos^2(\theta) - r^2\sin^2\theta = 1 - r^2.$

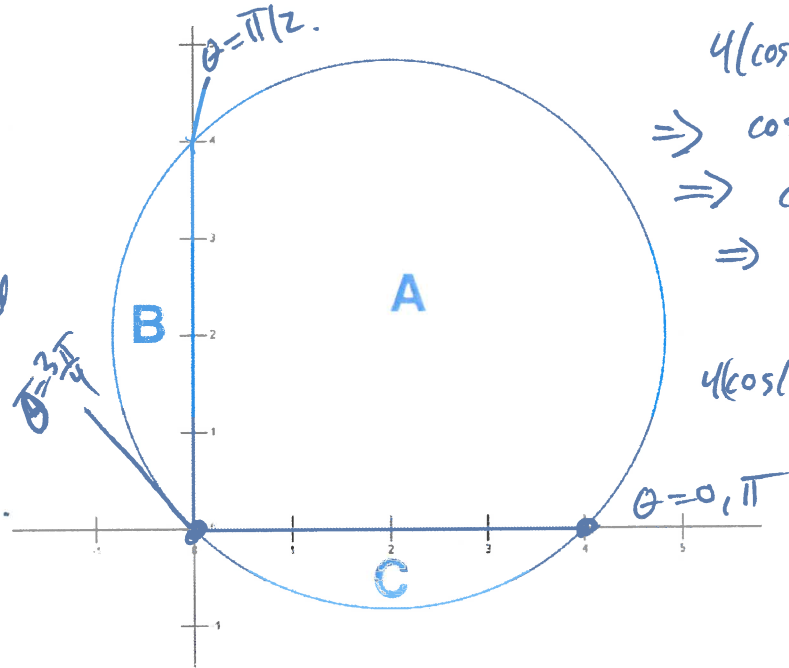
$$\iiint_W (x^2+y^2)\,dV = \int_0^{2\pi}\int_0^1\int_0^{1-r^2} r^2 \cdot r\, dz\,dr\,d\theta = \int_0^{2\pi}\int_0^1 \Big[zr^3\Big]_0^{1-r^2} dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^1 (r^3 - r^5)\,dr\,d\theta = \int_0^{2\pi}\left[\frac{r^4}{4} - \frac{r^6}{6}\right]_0^1 d\theta = \int_0^{2\pi}\underbrace{\left(\frac{1}{4} - \frac{1}{6}\right)}_{=\frac{3}{12}-\frac{2}{12}=\frac{1}{12}} d\theta$$

$$= \int_0^{2\pi} \frac{1}{12}\,d\theta = \left[\frac{\theta}{12}\right]_0^{2\pi} = \boxed{\frac{\pi}{6}}$$

5. (6) Consider the curve $r = 4(\cos(\theta) + \sin(\theta))$, graphed below for $0 \le \theta \le \pi$.

There are many ways to do this, some without any calculus at all! Any correct, justified answer gets full credit. I'll do the calculus solution here.



$4(\cos(\theta)+\sin(\theta)) = 0$

$\Rightarrow \cos(\theta)+\sin(\theta) = 0$

$\Rightarrow \cos(\theta) = -\sin(\theta)$

$\Rightarrow \theta = \frac{3\pi}{4}$

$4(\cos(\theta)+\sin(\theta)) = 4$

$\Rightarrow \theta = 0, \pi.$

Let A be the region that lies inside the curve with $x \ge 0$ and $y \ge 0$. Let B be the region bounded that lies inside the curve with $x \le 0$. Let C be the region that lies inside the curve with $y \le 0$.

(a) Find the area of the region A.

$$\text{Area}(A) = \iint_A 1\,dA = \int_0^{\pi/2}\int_0^{4(\cos(\theta)+\sin(\theta))} r\,dr\,d\theta = \frac{1}{2}\int_0^{\pi/2} \underbrace{(4\cos(\theta)+4\sin(\theta))^2}_{16\cos^2\theta + 32\cos\theta\sin\theta + 16\sin^2\theta}\,d\theta$$

$$= 8\int_0^{\pi/2}(1+2\cos(\theta)\sin(\theta))\,d\theta = 8\Big[\theta + \sin^2(\theta)\Big]_{\theta=0}^{\theta=\pi/2}$$

$$= 8\left(\left(\frac{\pi}{2} + \sin^2\left(\frac{\pi}{2}\right)\right) - \left(0 + \frac{\sin^2(0)}{2}\right)\right)$$

$$= 8\left(\frac{\pi}{2} + 1\right) = \boxed{4\pi + 8}$$

(b) Find the area of the region B.

Option 1: $\text{Area}(B) = \iint_B 1\,dA = \int_{\pi/2}^{3\pi/4} \underbrace{\int_0^{4\cos\theta + 4\sin\theta} r\,dr\,d\theta}_{\text{same as (a)}}$

$= 8\left[\theta + \sin^2(\theta)\right]_{\theta=\pi/2}^{\theta=3\pi/4}$

$= 8\left(\left(\frac{3\pi}{4} + \sin^2\left(\frac{3\pi}{4}\right)\right) - \left(\frac{\pi}{2} + \sin^2\left(\frac{\pi}{2}\right)\right)\right)$

$= 8\left(\frac{\pi}{4} + \frac{1}{2} - 1\right) = \boxed{(2\pi - 4)}$

Option 2: center of circle: $(2,2)$. radius $= 2\sqrt{2}$.

area $= \pi(4\sqrt{2})^2 = 8\pi$.

Since $\text{Area}(B) = \text{Area}(C)$, $\text{Area}(B) = \frac{1}{2}(8\pi - \text{Area}(A))$

$= \frac{1}{2}(8\pi - (4\pi + 8))$

$= 2\pi - 4$.

(c) Find the area of the region C.

Clearly, $\text{Area}(B) = \text{Area}(C)$,

so, $\boxed{2\pi - 4}$

6. (6) Let R be the region bounded by the curves $xy = 1$, $xy = 2$, $xy^2 = 1$, and $xy^2 = 2$. Use the transformation $u = xy$ and $v = xy^2$ to calculate $\iint_R y^2\, dA$.

Identical to HW 4, #41.

The region of integration is $\left\{\begin{array}{l} 1 \le xy \le 2 \\ 1 \le xy^2 \le 2 \end{array}\right\} = \left\{\begin{array}{l} 1 \le u \le 2 \\ 1 \le v \le 2. \end{array}\right.$

We need to solve for x and y:

$$u = xy \Rightarrow x = \frac{u}{y} \qquad \Rightarrow \frac{u}{y} = \frac{v}{y^2} \Rightarrow y^2 u = yv$$
$$v = xy^2 \Rightarrow x = \frac{v}{y^2} \qquad \Rightarrow y = \frac{v}{u}$$

So, $x = \frac{u}{v/u} = \frac{u^2}{v}.$

$$\mathrm{Jac}(G) = \begin{vmatrix} \frac{2u}{v} & -\frac{u^2}{v^2} \\ -\frac{v}{u^2} & \frac{1}{u} \end{vmatrix} = \frac{2}{v} - \frac{1}{v} = \frac{1}{v}.$$

$$\iint_R y^2\, dA = \int_1^2 \int_1^2 \frac{v^2}{u^2} \cdot \left|\frac{1}{v}\right| du\, dv = \int_1^2 \int_1^2 \frac{v}{u^2}\, du\, dv$$

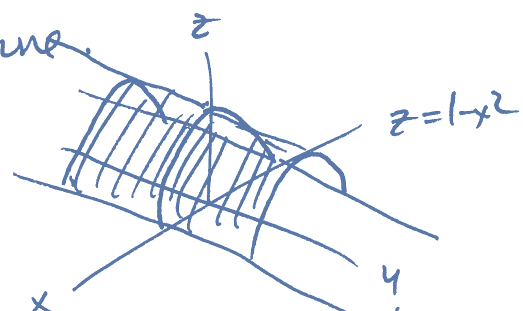
$$= \left(\int_1^2 \frac{1}{u^2}\, du\right)\left(\int_1^2 v\, dv\right) = \left[-\frac{1}{u}\right]_1^2 \cdot \left[\frac{v^2}{2}\right]_1^2 = \left(-\frac{1}{2} + 1\right)\left(2 - \frac{1}{2}\right)$$

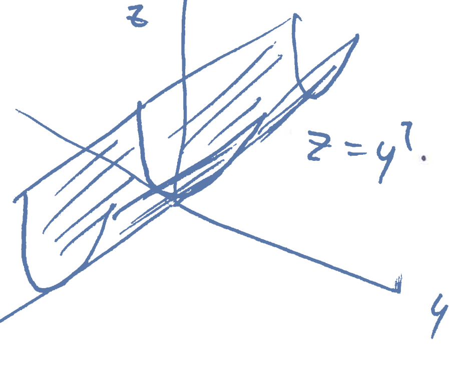
$$= \frac{1}{2} \cdot \frac{3}{2} = \boxed{\frac{3}{4}}$$

7. (6) Complete each question. (The two parts are unrelated.)

(a) Find the volume of the solid region bounded by the surfaces $z = y^2$ and $z = 1 - x^2$.

Hard to draw, but we don't need that good of a picture.





Where do they intersect?

$1-x^2 = y^2$

$1 = x^2+y^2$

circle of radius 1 at (0,0).

$z=y^2 \Rightarrow$ bottom

$z=1-x^2 \Rightarrow$ top

cylindrical $\rightarrow \displaystyle\int_0^{2\pi}\int_0^1\int_{r^2\sin^2\theta}^{1-r^2\cos^2\theta} r\,dz\,dr\,d\theta$

$$= \int_0^{2\pi}\int_0^1 r\left(-r^2\sin^2(\theta) - r^2\cos^2(\theta) + 1\right)dr\,d\theta = \int_0^{2\pi}\int_0^1 (r - r^3)\,dr\,d\theta$$

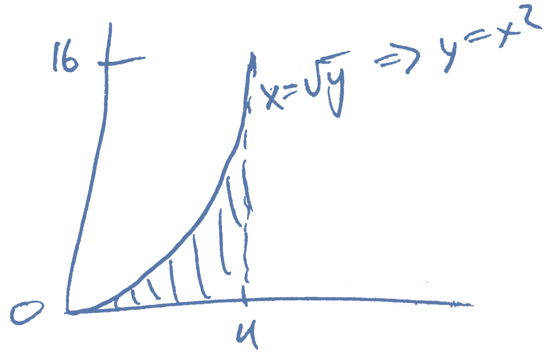
$$= \int_0^{2\pi}\left[\frac{r^2}{2} - \frac{r^4}{4}\right]_0^1 d\theta = \int_0^{2\pi}\frac{1}{4}\,d\theta = \boxed{\frac{\pi}{2}}$$

(b) Rewrite the integral with the order of integration reversed. *Do not evaluate the integral.*

$$\int_0^{16}\int_{\sqrt{y}}^{4}(3xy+y^2)\,dx\,dy$$

(from homework!)

Draw the region:



$$\int_0^4\int_0^{x^2}(3xy+y^2)\,dy\,dx$$

For the problems on this page, you may use **Clairaut's Theorem** from Math 8.
Clairaut's Theorem: If $f(x,y)$ has second partial derivatives that exist and are continuous, then

$$\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right).$$

In other words, the order in which you take the partial derivatives doesn't matter. The theorem holds for functions in three variables, too.

8. (0) **BONUS!** (3 points) Don't attempt this problem until you've finished the rest of the exam.

Let $\boldsymbol{F}$ be any vector field with components whose second partial derivatives all exist and are continuous. Prove that

$$\operatorname{div}(\operatorname{curl}(\boldsymbol{F})) = 0.$$

Proof: Let $\vec{F} = \langle F_1, F_2, F_3 \rangle$. Then $\operatorname{curl}(\vec{F}) = \begin{vmatrix} \vec{i} & \vec{j} & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}$

$$= \left\langle \frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}, \frac{\partial F_1}{\partial z} - \frac{\partial F_3}{\partial x}, \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right\rangle$$

So, $\operatorname{div}(\operatorname{curl}(\vec{F})) = \frac{\partial}{\partial x}\left(\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}\right) + \frac{\partial}{\partial y}\left(\frac{\partial F_1}{\partial z} - \frac{\partial F_3}{\partial x}\right) + \frac{\partial}{\partial z}\left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right)$

$$= \frac{\partial F_3}{\partial xy} - \frac{\partial F_2}{\partial xz} + \frac{\partial F_1}{\partial yz} - \frac{\partial F_3}{\partial yx} + \frac{\partial F_2}{\partial zx} - \frac{\partial F_1}{\partial zy} = 0. \text{ QED}$$

$=$, $=$, $=$, by Clairaut's Thm

9. (0) **BONUS!** (3 points) Don't attempt this problem until you've finished the rest of the exam.

Let $f(x,y,z)$ be any scalar function whose second partial derivatives all exist and are continuous. Prove that

$$\operatorname{curl}(\nabla f) = \boldsymbol{0}.$$

Proof: $\nabla f = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right\rangle$.

So, $\operatorname{curl}(\vec{F}) = \begin{vmatrix} \vec{i} & \vec{j} & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} & \frac{\partial f}{\partial z} \end{vmatrix}$

$$= \left\langle \frac{\partial f}{\partial yz} - \frac{\partial f}{\partial zy}, \frac{\partial f}{\partial zx} - \frac{\partial f}{\partial xz}, \frac{\partial f}{\partial xy} - \frac{\partial f}{\partial yx} \right\rangle = \langle 0, 0, 0 \rangle = \vec{0}$$

by Clairaut's Theorem QED.