Math 13 Fall 2009 practice exam

1. Find the equation of the plane containing the two lines

$$x = 1 + t \qquad y = 1 + 2t \qquad z = 1 + 3t$$

and

$$x = -3 + 2t \qquad y = -2 - t \qquad z = -4 - t$$

2. Express the vector $\langle 5, 1, 2 \rangle$ as a sum of a vector parallel to $\langle 1, 2, 2 \rangle$ and a vector orthogonal to $\langle 1, 2, 2 \rangle$.

3. Assume that the height of a mountain is given by the graph $z = f(x, y)$.
a) Suppose that at the point P the slope due east is $\frac{1}{3}$ and the slope due north is $-\frac{1}{3}$. In what direction should you head in order to ascend most rapidly? In what direction should you head to descend most rapidly?
b) Suppose that at the point Q the slope in the direction $\langle \frac{3}{5}, \frac{4}{5} \rangle$ is 2 and in the direction $\langle 1, 0 \rangle$ it is 3. Find

$$\frac{\partial f}{\partial y}$$

at the point Q.

4. Let $f(x, y) = 2 + \ln(x^2 + y^2)$.
a) Sketch at least three level curves of f.
b) Find the tangent plane to the graph of f at the point $(x, y) = (1, 1)$.
c) Find the tangent line at $(1, 1)$ to the level curve of f that passes through that point.

5. Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ be given by $f(x, y) = (x^2 y + y^2, x + 2xy)$. Let $g : \mathbb{R}^2 \to \mathbb{R}^2$ be given by $g(s, t) = (s - t, s^2 - t^3)$.
a) Find $(f \circ g)'(3, 2)$.
b) Writing $f(x, y) = (u, v)$ read of from part a) the partial

$$\frac{\partial v}{\partial s}(3, 2)$$

6. Compute the following interated integrals.

a)

$$\int_{-1}^{2} \int_{1}^{4} x^2 \ln(y) dy dx$$

b)

$$\int_{2}^{5} \int_{x-1}^{x^2} \cos(x) y dy dx$$

c)

$$\int_{-1}^{1} \int_{0}^{\sqrt{1-x^2}} \frac{y}{1+x} dy dx$$

7. Consider the curve $r(t) = (t^2, t, t^3 - 1)$. Find all points on this curve at which the tangent line to the curve is parallel to the plane $x - y + z = 0$. (You can either specify the points or just give the values of t that give the points).