Math 13
Calculus of Vector-Valued Functions
Fall 2005
Assignment 1
Due September 28, 2005

Note: Please Show All of Your Work.

Section 1.6: 36
Section 2.2: 23
Section 2.3: 34

1. On p. 54 of your text a mapping $T : \mathbb{R}^n \to \mathbb{R}^m$ is defined to be linear if it is of the form

$$T(\mathbf{v}) = A\mathbf{v},$$

where $A = (a_{ij})$ is an $m \times n$ matrix. Another way of defining a linear map is as follows.

Definition 1. *A map $T : \mathbb{R}^n \to \mathbb{R}^m$ is said to be* **linear** *if for any $\mathbf{v}, \mathbf{w} \in \mathbb{R}^n$ and $c \in \mathbb{R}$ we have*

(a) $T(\mathbf{v} + \mathbf{w}) = T(\mathbf{v}) + T(\mathbf{w})$.

(b) $T(c\mathbf{v}) = cT(\mathbf{v})$

Using this definition verify directly that the map $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by $T(x, y) = (2x + y, x + y)$ is linear. What is the matrix representation of this map?

(**Note:** It might be instructive to show that these two definitions of linear maps are equivalent.)

2. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be a 2×2 matrix and $B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$ be a 3×3 matrix. Verify by direct computation that

(a) $\det(A) = \det(A^t)$, where $A^t = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$ is the transpose of A.

(b) $\det(B) = \det(B^t)$, where $B^t = \begin{bmatrix} b_{11} & b_{21} & b_{31} \\ b_{12} & b_{22} & b_{32} \\ b_{13} & b_{23} & b_{33} \end{bmatrix}$ is the transpose of B.

3. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x, y) = x^2 + xy - 3y^2$. Verify using the definition of the partial derivative that $\frac{\partial f}{\partial x}(x, y) = 2x + y$ and $\frac{\partial f}{\partial y}(x, y) = x - 6y$