(1) Find the center of mass of the shape located inside of the cylinder $x^2 + y^2 = 1$ that is bounded from below by the plane $z = -1$ and from above by the plane $z = 2$. The density is $\rho(x, y, z) = x^2 + y^2$.

(2) Let $\mathbf{x}(t) = (3\cos t, 3\sin t, 3)$, $0 \le t \le \pi$, be a path in $\mathbb{R}^2$. Let $f(x, y) = 2x^2 + 2y^2$ be a function and let $\mathbf{F}(x, y, z) = 2x\mathbf{i} + 2y\mathbf{j} + 2z\mathbf{k}$ be a vector field on $\mathbb{R}^3$.

a: Find $\int_{\mathbf{x}} f ds$.

b: Find $\int_{\mathbf{x}} \mathbf{F} \cdot \mathbf{ds}$.

(3) Find the work done by the force field $\mathbf{F}(x, y, z) = xz\mathbf{i} + e^{\cos z}\mathbf{j} - 3z\mathbf{k}$ on the particle that moved along the path $c(t) = (t^2, 117, t^3)$ for $0 \le t \le 1$.

(4) Is the following vector field $\mathbf{F}(x, y, z) = 3x^2 \cos y\mathbf{i} + (z^2 - x^3 \sin y)\mathbf{j} + 2yz\mathbf{k}$ a conservative vector field? If yes, then use the line integral to find the potential function.

(5) Let D be the part of the disk $x^2 + y^2 \le 1$ located above the x-axis with a positively oriented boundary ∂D. Calculate $\oint_{\partial D}(y^2 + 3x^3)dy + (-3y^3 + \sin x)dx$. **Hint: Be careful, look at the integral twice.**

(6) Let $\mathbf{F}(x, y) = (7x + \sin y)\mathbf{i} + (e^x - 6y)\mathbf{j}$ be a vector field, and let $\mathbf{n}$ be the outward unit normal to the positively oriented circle $C = \{(x, y) | x^2 + y^2 = 9\}$. Calculate the flux integral $\oint_C \mathbf{F} \cdot \mathbf{n} ds$.

(7) Consider the double integral $\int_0^1 \int_0^{\sqrt{1-y}} e^{3x-x^3}\, dx\, dy$.

(a) Sketch the region of integration.

(b) Evaluate the integral.

(8) Let B be the portion of the solid ball $x^2 + y^2 + z^2 \le a^2$ in the first octant as shown.

(a) Set up, but do not evaluate, three triple integrals for $\iiint_B z\, dV$. Use cartesian coordinates in one integral, cylindrical coordinates in another, and spherical coordinates in the third.

(b) Evaluate one of the three integrals in part (a).

(9) Let D be the region in the xy-plane bounded by the lines $y = -2x + 4$, $y = -2x + 7$, $y = x - 2$ and $y = x + 1$. Evaluate $\iint_D (x - y)\sqrt{2x + y}\, dx\, dy$ by changing variables.

(10) (no work required) Set up, but do not evaluate, the integrals for the volume of the solid bounded by $z = 1$, $y = 0$, and $z = x^2 + y$ using the two orders of integration indicated below.

$$\int \int \int \quad dx\, dy\, dz \qquad\qquad \int \int \int \quad dz\, dy\, dx$$

(11) Find the centroid of the quarter of disk of radius a located in the first quadrant (i.e. above the x-axis and to the right from the y-axis).