(1) Find the arc length parameterization of the curve in $\mathbb{R}^3$ $\mathbf{r}(t) = (\cos(2t), \sin(2t), \frac{2}{3}t^{\frac{3}{2}})$, $0 \leq t \leq 3$.

(2) Find the volume of the "cap" of the ball

$$\{(x, y, z,)|x^2 + y^2 + z^2 \leq 4\}$$

located above the plane $z = 1$. Hint: if you slice the cap in one of the directions you get disks.

(3) a. Calculate curl(F) of the vector field $\mathbf{F}(x, y, z) = \sin(x^3)\mathbf{i} + e^{xz}\mathbf{j} + e^{z^2}\mathbf{k}$.

b. Calculate the divergence div $\mathbf{G}$ of the 3-dimensional vector field $\mathbf{G}(x_1, x_2, x_3) = \cos(e_2^{\pi})\mathbf{i} + x_3^3\mathbf{j} + x_1^{2004}\mathbf{k}$.

(4) Is the vector field $F(x, y, z) = (3xy, x+y, z^3)$ equal to ∇f for some function $f : \mathbb{R}^3 \to \mathbb{R}^1$.

(5) Calculate $\int_{\pi}^{2\pi} \int_1^3 \left(\frac{\sin x}{x} + 3\right) \cos y dx dy$. Hint: you might want to interchange the order of integration.

(6) (a) Let S be the sphere of radius 7 centered at the orgin. Find an equation for S in **cylindrical** coordinates.
(b) Convert the equation $\rho = \sin\varphi \sin\theta$ in **spherical** cooridnates to one involving cartesian coordinates.

(7) (no work required) Match each set below with its verbal description on the right. One of the sets does not match any of the given descriptions. Say which set this is, and give a brief description of it.

_ $\{(r, \theta, z) : 0 \leq r \leq 3,\ \frac{\pi}{2} \leq \theta \leq \frac{3\pi}{4},\ 0 \leq z \leq 8\}$

_ $\{(\rho, \varphi, \theta) : \rho = 1,\ 0 \leq \varphi \leq \pi,\ \theta = \pi\}$

_ $\{(\rho, \varphi, \theta) : \varphi = \frac{3\pi}{4},\ \rho = \sqrt{2}\}$

_ $\{(r, \theta, z) : 0 \leq r \leq 3,\ 0 \leq z \leq 8\}$

_ $\{(\rho, \varphi, \theta) : 0 \leq \rho \leq 1\}$

A. a semi-circle in the xz-plane

B. a soup can with its contents

C. a circle centered at $(x, y, z) = (0, 0, -1)$

D. a circle centered at $(x, y, z) = (0, 0, 0)$

E. a sphere centered at (1,0,0)

F. a sort of wedge cut from a cylinder

G. a sort of wedge cut from asphere

(8) Either explain why the limit

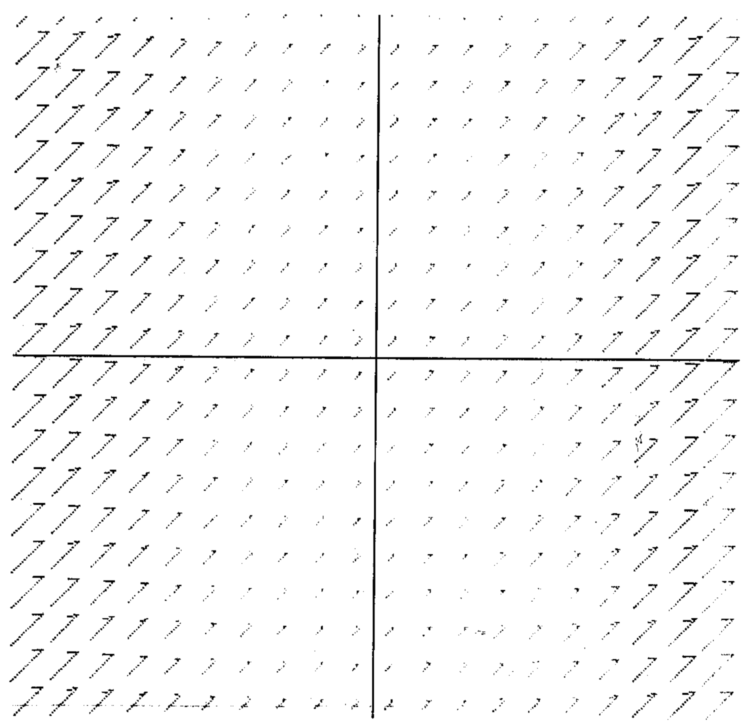
$$\lim_{(x,y)\to(0,0)} \frac{5x^3 - 2y^4}{x^2 + y^2}$$

exists and find its value, or show that it does not exist.

(9) (no work required) Consider the vector field $\mathbf{F}$ shown below. All the vectors $\mathbf{F}(x, y)$ of this vector field are parallel, but their lengths vary as shown.

(a) Is $\operatorname{div}\mathbf{F}$ at $(1, 1)$ positive, negative or zero?

(b) Is $\operatorname{div}\mathbf{F}$ at $(-1, -1)$ positive, negative or zero?



(10) Consider the function $f\colon \mathbb{R}^2 \to \mathbb{R}^3$ defined by $f(x,y) = (x^2, x^3y, x^4y^3)$. Find the matrix $Df(x,y)$, and find $Df(1,1)$.

(11) Assume $f\colon \mathbb{R}^3 \to \mathbb{R}^2$ and $g\colon \mathbb{R}^2 \to \mathbb{R}^2$ are differentiable functions satisfying $f(1,2) = (-1,-2)$, $f(3,4) = (-3,-4)$, $g(1,2) = (3,4)$, $g(-1,-2) = (-3,-4)$, $Df(3,4) = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$, $Df(1,2) = \begin{pmatrix} 0 & 1 & 2 \\ 2 & 1 & 0 \end{pmatrix}$, $Dg(1,2) = \begin{pmatrix} 3 & 4 \\ -4 & -3 \end{pmatrix}$, and $Dg(-1,-2) = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$. If enough information is given, find $D(g \circ f)(1,2)$ and express your answer as a single matrix. In case not enough information is given, say specifically what additional information is needed to find $D(g \circ f)(1,2)$.