

GREEN'S THEOREM AND ITS REFORMULATIONS

Let D be a closed bounded region in \mathbb{R}^2 such that its boundary ∂D consists of finitely many simple closed curves that are **oriented** in such a way that D is on the **left** as one traverses ∂D . Let $\mathbf{F}(x, y) = M(x, y)\mathbf{i} + N(x, y)\mathbf{j}$ be a vector field of class C^1 .

1. **(Green's Theorem)**
$$\oint_{\partial D} M dx + N dy = \iint_D \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dA.$$

2. **(Vector reformulation)**
$$\oint_{\partial D} \mathbf{F} \cdot d\mathbf{s} = \iint_D (\nabla \times \mathbf{F}) \cdot \mathbf{k} dA.$$

3. **(Divergence Theorem in the plane)** If \mathbf{n} is the outward unit normal vector to D , then
$$\oint_{\partial D} \mathbf{F} \cdot \mathbf{n} ds = \iint_D \nabla \cdot \mathbf{F} dA.$$