Green's Theorem and its Reformulations

Let D be a closed bounded region in $\mathbb{R}^2$ such that its boundary ∂D consists of finitely many simple closed curves that are **oriented** in such a way that D in on the **left** as one traverses ∂D. Let $\mathbf{F}(x, y) = M(x, y)\mathbf{i} + N(x, y)\mathbf{j}$ be a vector field of class C^1.

1. **(Green's Theorem)** $\displaystyle\oint_{\partial D} M\,dx + N\,dy = \iint_D \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA.$

2. **(Vector reformulation)** $\displaystyle\oint_{\partial D} \mathbf{F} \cdot d\mathbf{s} = \iint_D (\nabla \times \mathbf{F}) \cdot \mathbf{k}\,dA.$

3. **(Divergence Theorem in the plane)** **If** $\mathbf{n}$ is the outward unit normal vector to D, **then** $\displaystyle\oint_{\partial D} \mathbf{F} \cdot \mathbf{n}\,ds = \iint_D \nabla \cdot \mathbf{F}\,dA.$