Green's, Stokes's, and Gauss's Theorems

Let D be a closed bounded region in $\mathbb{R}^2$ such that its boundary ∂D consists of finitely many simple closed curves that are **oriented** in such a way that D in on the **left** as one traverses ∂D. Let $\mathbf{F}(x,y) = M(x,y)\mathbf{i} + N(x,y)\mathbf{j}$ be a vector field of class C^1.

1. **(Green's Theorem)** $\displaystyle\oint_{\partial D} M\,dx + N\,dy = \iint\limits_D \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA.$

2. **(Vector form of Green's Theorem)** $\displaystyle\oint_{\partial D} \mathbf{F}\cdot d\mathbf{s} = \iint\limits_D (\nabla\times\mathbf{F})\cdot\mathbf{k}\,dA.$

3. **(Divergence Theorem in the plane)** **<u>If</u>** $\mathbf{n}$ is the outward unit normal vector to D, **<u>then</u>** $\displaystyle\oint_{\partial D} \mathbf{F}\cdot\mathbf{n}\,ds = \iint\limits_D \nabla\cdot\mathbf{F}\,dA.$

Let S be a bounded, oriented surface in $\mathbb{R}^3$ such that its boundary ∂S consists of finitely many simple closed curves that are oriented **consistently** with S. Let $\mathbf{F}$ be a vector field of class C^1.

4. **(Stokes's Theorem)** $\displaystyle\oint_{\partial S} \mathbf{F}\cdot d\mathbf{s} = \iint\limits_S (\nabla\times\mathbf{F})\cdot d\mathbf{S}.$

Let W be a bounded solid region in $\mathbb{R}^3$ such that its boundary ∂W consists of finitely many closed orientable surfaces that are oriented by unit normals $\mathbf{n}$ pointing **away** from W. Let $\mathbf{F}$ be a vector field of class C^1.

5. **(Gauss's Theorem)** $\displaystyle\oiint\limits_{\partial W} \mathbf{F}\cdot d\mathbf{S} = \oiint\limits_{\partial W} (\mathbf{F}\cdot\mathbf{n})dS = \iiint\limits_W \nabla\cdot\mathbf{F}\,dV$