

# CHANGE OF VARIABLES FORMULAS

## Double integrals:

$$\iint_D f \, dA = \iint_D f(x, y) \, dx \, dy = \iint_{D^*} f(x(u, v), y(u, v)) \left| \frac{\partial(x, y)}{\partial(u, v)} \right| \, du \, dv = \iint_{D^*} f(u, v) \, dA^*$$

## Area elements:

Cartesian coordinates:  $dA = dx \, dy$

Linear transformation

$x = au + bv, y = cu + dv:$   $dA = |ad - bc| \, du \, dv$

Polar coordinates:  $dA = r \, dr \, d\theta$

General case:  $dA = \left| \frac{\partial(x, y)}{\partial(u, v)} \right| \, du \, dv$

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## Triple integrals:

$$\iiint_W f \, dV = \iiint_W f(x, y, z) \, dx \, dy \, dz =$$

$$\iiint_{W^*} f(x(u, v, w), y(u, v, w), z(u, v, w)) \left| \frac{\partial(x, y, z)}{\partial(u, v, w)} \right| \, du \, dv \, dw = \iiint_{W^*} f(u, v, w) \, dV^*$$

## Volume elements:

Cartesian coordinates:  $dV = dx \, dy \, dz$

Cylindrical coordinates:  $dV = r \, dr \, d\theta \, dz$

Spherical coordinates:  $dV = \rho^2 \sin \varphi \, d\rho \, d\varphi \, d\theta$

General case:  $dV = \left| \frac{\partial(x, y, z)}{\partial(u, v, w)} \right| \, du \, dv \, dw$