MATH 11: MULTIVARIABLE CALCULUS
FALL 2018
HOMEWORK #9

Please turn in your completed homework assignment by leaving it in the boxes labeled "Math 11" in the hallway outside of Kemeny 105 anytime before 3:30 p.m. on **Tuesday, November 13**.

Problem 1. Find $\iint_S \left\langle \frac{x}{(x^2+y^2+z^2)^{3/2}}, \frac{y}{(x^2+y^2+z^2)^{3/2}}, \frac{z}{(x^2+y^2+z^2)^{3/2}} \right\rangle$ by parametrizing the surface and directly evaluating the surface integral, where

(a) S is the sphere $x^2+y^2+z^2=a^2$, oriented with normal vector pointing outward.

(b) S is the disc $x^2+y^2 \leq 1$ in the plane $z=1$, oriented with normal vector pointing upward.

Problem 2. Compute the curl of the vector field
$F(x,y,z) = \langle e^x + y^2, y^2+z^2, \sin(z)+x^2 \rangle$, and use Stokes' Theorem to find

(a) $\int_\gamma F \cdot d\vec{r}$, where γ is the ellipse $3x^2+2y^2=6$ in the plane $z=1$, oriented counterclockwise as viewed from above. Hint: γ is the boundary of a surface contained in the plane $z=1$.

(b) $\iint_S \operatorname{curl}(F) \cdot \vec{n}\, dS$, where S is the portion of the paraboloid $z = 7-3x^2-2y^2$ above the plane $z=1$, oriented with normal vector pointing upward.

Problem 3. Compute the divergence of the vector field
$F(x,y,z) = \left\langle \frac{x}{(x^2+y^2+z^2)^{3/2}}, \frac{y}{(x^2+y^2+z^2)^{3/2}}, \frac{z}{(x^2+y^2+z^2)^{3/2}} \right\rangle$.
Use this, the divergence theorem, the results of problem 1 if applicable, and symmetry if it helps, to find $\iint_S F \cdot \vec{n}\, dS$ without directly evaluating any additional surface integrals, where

(a) S is a sphere of radius 1 with center $(-1,1,1)$. Hint: Are there any points at which $\nabla \cdot F$ is problematic? Are they inside or on S?

(b) S is a sphere of radius 3 with center $(-1,1,1)$. Hint: If you are not sure how to get started on this one, look at the subsection "Application to Electrostatic Fields" of Section 6.8 of the textbook.

(c) S is the cylindrical surface $x^2+y^2=1$, $-1 \leq z \leq 1$, oriented with normal vector pointing away from the z-axis. Hint: S is part of the surface of the three-dimensional region given by $x^2+y^2+z^2 \leq 1$, $-1 \leq z \leq 1$, oriented with normal vector pointing outward. The strategy from part (b) may be helpful.

Date: Due **Tuesday, November 13**, 3:30 p.m.