MATH 11: MULTIVARIABLE CALCULUS
FALL 2018
HOMEWORK #7

Please turn in your completed homework assignment by leaving it in the boxes labeled "Math 11" in the hallway outside of Kemeny 105 anytime before 3:30 p.m. on **Wednesday, October 31**.

Problem 1. Let C be the union of the straight line starting at $(0,0)$ and ending at $(2,1)$ with the quarter circle from $(2,1)$ to $(3,0)$ with center $(2,0)$ traversed clockwise.

(a) Compute $\int_C xy\,ds$.

(b) Compute $\int_C y\,dx - x\,dy$.

(c) Describe how your answers to (a) and (b) would change if C were replaced with $-C$, that is, the same path traversed in the opposite sense.

Problem 2.

(a) Consider the vector field

$$\mathbf{F} = \left\langle \frac{x}{x^2+y^2}, \frac{y}{x^2+y^2} \right\rangle,$$

and the curve C parametrized as $\mathbf{r}(t) = \langle t, \sin \pi t \rangle$, with $1 \leq t \leq 2$. Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$. Explain how you got your answer.

(b) The vector field $\mathbf{F}$ is not continuous at $(x,y) = (0,0)$. Nevertheless, $\mathbf{F}$ is a conservative vector field. Show this directly by finding a potential function $f(x,y)$ for $\mathbf{F}$.

(c) Let C be the ellipse $\frac{x^2}{4} + \frac{y^2}{16} = 1$, with counter clockwise orientation. Observe that the ellipse circles around the singular point $(0,0)$. Find the value of $\oint_C \mathbf{F} \cdot d\mathbf{r}$. Justify your answer.

Date: Due **Wednesday, October 31**, 3:30 p.m.