Math 11
Fall 2018
Practice Final Exam

Disclaimer: This practice exam should give you an idea of the sort of questions we may ask on the actual exam. Since the practice exam (like the real exam) is not long enough to cover everything we studied, there may be topics on the real exam that are not on the practice exam, and vice versa. Anything covered in assigned reading, class, WeBWorK, or written homework is fair game.

Advice: A good way to use the practice exam is to first study and prepare for the exam. Then take a couple of hours, sit in a quiet place, and take the practice exam as if it were the real exam. That should tell you which areas you should study further.

About the real exam: There may be short answer questions that will be graded only on the answer, and there will definitely be questions on which we grade on your work, your explanations, as well as the answer.

1. TRUE or FALSE? (No partial credit; you need not show your work.)

   (a) If S is the unit sphere centered at the origin, oriented outward and the flux integral $\iint_S \vec{F} \cdot d\vec{A}$ is zero, then $\vec{F} = \vec{0}$.

   (b) If S_1 is a rectangle with area 1 and S_2 is a rectangle with area 2, then $2 \iint_{S_1} \vec{F} \cdot d\vec{A} = \iint_{S_2} \vec{F} \cdot d\vec{A}$.

   (c) If S is the unit sphere centered at the origin, oriented outward and $\vec{F} = x\vec{i} + y\vec{j} + z\vec{k}$, then the flux integral $\iint_S \vec{F} \cdot d\vec{A}$ is positive.

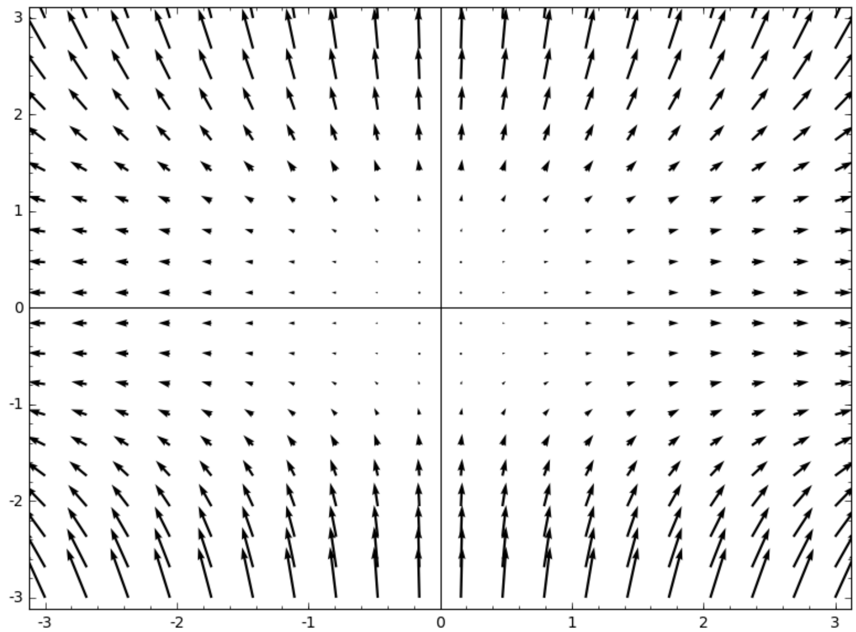
   (d) If S is an open-ended circular cylinder centered about the z-axis, oriented away from the z-axis, and $\vec{F} = \langle 3, -2, 6 \rangle$, then the flux of $\vec{F}$ through S is zero.

   (e) If $\mathbf{u}$ and $\mathbf{v}$ are non-zero and not parallel, the angle θ from $\mathbf{u}$ to $\mathbf{v}$ is equal to $\cot^{-1}\left(\frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u} \times \mathbf{v}\|}\right)$.

   (f) If $\mathbf{r}(t)$ is twice differentiable and $\|\mathbf{r}(t)\| = c > 0$ is a constant then $\mathbf{r}'(t)$ is orthogonal to $\mathbf{r}(t)$ for all t and $\mathbf{r}''(t) \cdot \mathbf{r}(t) < 0$ for all t.

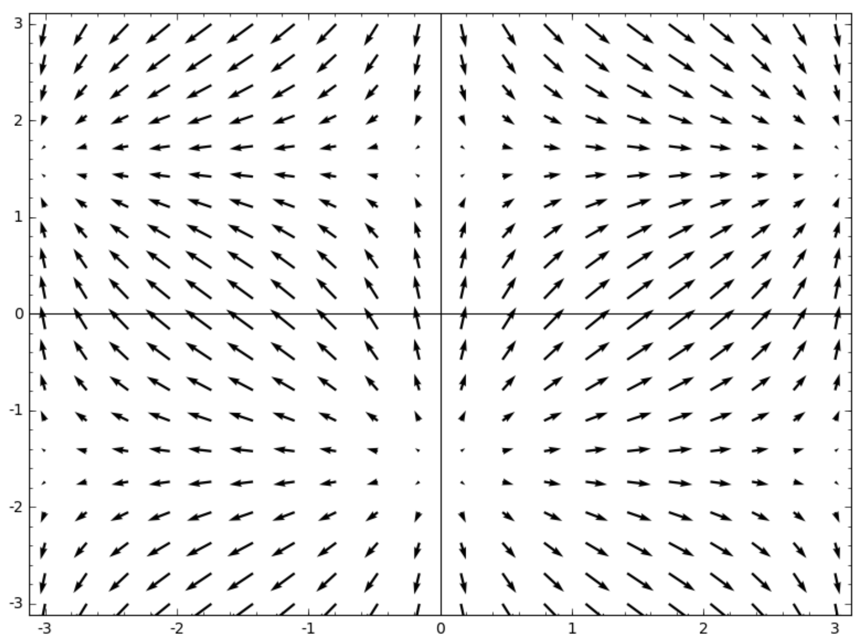
   (g) The function $u(x, y) = e^{-3x} \cos(3y)$ satisfies Laplace's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.

2. A vector field $\vec{F} = \langle f(x), g(y), 0\rangle$ is plotted on the xy-plane below.



Determine the sign of the flux across the parametrized surface $\vec{r}(u, v) = \langle 2\cos v, 2\sin v, u\rangle$, $0 \leq u \leq 1, -\pi/2 \leq v \leq \pi/2$.

3. The vector field $\vec{F}$ is plotted below.



(a) Is the divergence of $\vec{F}$ positive, negative, or zero at $(0, -1.5)$ and $(-2, 1)$?

(b) Is the curl of $\vec{F}$ positive, negative, or zero at $(1, 1)$?

4. Use change of variables to evaluate

$$\iint_D (x - 2y)\, dx\, dy$$

where D is the square with vertices $(0, 0)$, $(3, -1)$, $(4, 2)$, and $(1, 3)$.

5. Let $\mathbf{F}$ be the vector field $\mathbf{F} = \langle \sqrt{\sin x} + y^2, x^2 y \rangle$ and let C be the triangle with vertices $(0, 0)$, $(2, 0)$, and $(0, 3)$, oriented counterclockwise. Compute the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$.

6. An object moves according to parametric equations $x = \cos^2 t, y = 2t^2, z = \sin^2 t$. Find the following, as functions of time.

   (a) The position vector $\mathbf{r}(t)$.

   (b) the velocity vector $\mathbf{v}(t)$.

   (c) The acceleration vector $\mathbf{a}(t)$.

   (d) The speed of the object.

   (e) The tangential component of acceleration.

   (f) The normal component of acceleration.

   (g) The point on the curve which is half the distance along the curve between the points $\mathbf{r}(0)$ and $\mathbf{r}\left(\frac{\pi}{2}\right)$.

7. Let $f(x, y) = \frac{1}{2}x^2 + \frac{1}{2}y^2 - \ln(xy)$ where the domain of f is $\{(x, y) \mid x > 0, y > 0\}$.

   (a) Find all critical points of f.

   (b) Classify each critical point of f as either a local maximum, local minimum, or saddle point.

8. Let S be the surface $z = \sin x \cos y$, $0 \le x \le \pi, -\pi/2 \le y \le \pi/2$, oriented upward.

   (a) Parametrize the surface as $\vec{r}(u, v)$.

   (b) Find a parametrization for $-S$ (the surface with opposite orientation).

   (c) Set up the surface area integral for S.

   (d) Set up the surface integral of $f(x, y, z) = xyz$ over S.

   (e) Set up the surface integral of $\vec{F}(x, y, z) = \langle x, y, z \rangle$ over S.

9. Let S be the torus parametrized by

$$\mathbf{r}(u, v) = \langle (a + b\cos v)\cos u, (a + b\cos v)\sin u, b\sin v \rangle$$

where $0 \le u \le 2\pi$, $0 \le v \le 2\pi$ and $0 < b < a$. Use the parametrization to find the surface area of S.

10. Let C be the positively oriented boundary of the surface S defined by $z = 2 - x^2 - y^2$, where $0 \leq x \leq 1$, $0 \leq y$, and $0 \leq z$. Let $\mathbf{F}$ be the vector field $\mathbf{F} = \langle xyz, 0, xyz \rangle$.

    (a) Directly compute the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$.

    (b) Use Stokes' Theorem to find $\iint_S \text{curl}\mathbf{F} \cdot d\mathbf{S}$, where the surface is oriented so that the normals have a positive dot product with $\mathbf{k}$.

    (c) Using your answer from part (b), show that $\mathbf{F}$ is not conservative.

11. Use the Divergence Theorem to compute the surface integral $\iint_S \mathbf{F} \cdot \mathrm{d}\mathbf{S}$, where $\mathbf{F} = \langle x, y^2, z^3 \rangle$ and S is the boundary of the pyramid with vertices $(0, 0, 0)$, $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$.