Math 11
Fall 2018
Practice Exam II

Disclaimer: This practice exam should give you an idea of the sort of questions we may ask on the actual exam. Since the practice exam (like the real exam) is not long enough to cover everything we studied, there may be topics on the real exam that are not on the practice exam, and vice versa. Anything covered in assigned reading, class, WeBWorK, or written homework is fair game.

Advice: A good way to use the practice exam is to first study and prepare for the exam. Then take a couple of hours, sit in a quiet place, and take the practice exam as if it were the real exam. That should tell you which areas you should study further.

About the real exam: There may be short answer questions that will be graded only on the answer, and there will definitely be questions on which we grade on your work, your explanations, as well as the answer.

1. TRUE or FALSE? (No partial credit; you need not show your work.)

   (a) If $f(x, y)$ is differentiable at (a, b) and $f_x(a, b) = f_y(a, b)$, then all directional derivatives of f are equal to each other.

   (b) If $z = f(x, y)$ is a plane, then at each point (a, b) an equation of the tangent plane is $z = f(x, y)$.

   (c) If f_{xx} and f_{xy} have opposite signs at a critical point, then f has a saddle at that point.

   (d) If f has a local maximum at (a, b) and a local minimum at (c, d), then f must have a saddle point at some point (s, t).

2. Find all extrema of the function $f(x, y) = x^2 + y^2$ subject to the constraint $x^4 + y^4 = 1$.

3. The cylindrical coordinate integral

$$\int_0^{2\pi} \int_0^{\frac{2}{\sqrt{3}}} \int_2^{\sqrt{16-r^2}} r \, dz \, dr \, d\theta + \int_0^{2\pi} \int_{\frac{2}{\sqrt{3}}}^{2} \int_{\sqrt{3}r}^{\sqrt{16-r^2}} r \, dz \, dr \, d\theta$$

   describes the volume of a cupcake. Convert the integral to spherical coordinates, and then evaluate the integral.

4. Let $f(x, y) = x + y^2$ and $\vec{r}(t) = \langle \sin t, \cos t \rangle$.

   (a) Use the chain rule to find df/dt when $t = \pi/4$.

   (b) At the point $(1, 1)$, in which direction does f increase the fastest?

(c) How fast does f increase at $(1,1)$ in the direction $\langle 1,1\rangle$?

(d) Find a direction in which f does not increase nor decrease at $(1,1)$.

5. Evaluate
$$\int_0^1 \int_{-\sqrt{y}}^{\sqrt{y}} e^x \, dx \, dy.$$

6. Use a triple integral to determine the volume of a tetrahedron with vertices $(1,0,1)$, $(0,1,1)$, $(1,1,0)$, and $(1,1,2)$. Chose one other order of integration, and set up an integral for the volume in this order as well.

7. Find all critical points of the function $f(x,y) = x^3 - 6xy + 8y^3$ and classify them.