Math 11
Fall 2018
Practice Exam I

Disclaimer: This practice exam should give you an idea of the sort of questions we may ask on the actual exam. Since the practice exam (like the real exam) is not long enough to cover everything we studied, there may be topics on the real exam that are not on the practice exam, and vice versa. Anything covered in assigned reading, class, WeBWorK, or written homework is fair game.

Advice: A good way to use the practice exam is to first study and prepare for the exam. Then take a couple of hours, sit in a quiet place, and take the practice exam as if it were the real exam. That should tell you which areas you should study further.

About the real exam: There may be short answer questions that will be graded only on the answer, and there will definitely be questions on which we grade on your work, your explanations, as well as the answer.

1. TRUE or FALSE? (No partial credit; you need not show your work.)

   (a) If $f(x, y)$ is a symmetric function (i.e. $f(x, y) = f(y, x)$), then the function $f_x + f_y$ is also symmetric.

   (b) For any vectors $\vec{w}$ and $\vec{v}$ we have $\vec{w} \times \text{proj}_{\vec{w}}(\vec{v}) = \vec{0}$ (where $\text{proj}_{\vec{w}}(\vec{v})$ denotes the vector projection).

   (c) If $|\mathbf{r}'(t)|$ is constant, then $\mathbf{r}(t)$ parametrizes a line.

2. Does the limit
$$\lim_{(x,y)\to(0,0)} \frac{x^4 - y^4}{x^2 - y^2}$$
exist? Justify your answer.

3. Does the limit
$$\lim_{(x,y)\to(0,0)} \frac{x}{x + y^2}$$
exist? Justify your answer.

4. Let A, B, and C be points on a triangle in $\mathbb{R}^3$ such that AB is the diameter of a circle containing the point C. Use the dot product to show that the triangle is a right triangle.

5. Find the distance between the parallel planes $2x - y + 3z = 5$, $4x - 2y + 6z - 9 = 0$.

6. Find all points (x, y) at which the graphs of the functions $f(x, y) = x^2 - 3y^2$ and $g(x, y) = 2x + 2y^3$ have parallel tangent planes.

7. Find all points (x_0, y_0) such that the plane tangent to the surface $z = \sin(xy)$ at (x_0, y_0) is horizontal. Plot these points on the xy-plane.

8. Find the arc length for the curve parametrized by the vector-valued function $\mathbf{r}(t) = \langle \sin(t^3), 2t^3, \cos(t^3) \rangle$, $-1 \leq t \leq 1$. Find the velocity vector at the point $(0, 0, 1)$.

9. Let $f(x, y) = 3x^2y - 2y^2$. Determine $f_x(x, y)$, $f_y(x, y)$, $f_{xx}(x, y)$, and $f_{xy}(x, y)$. Find an equation for the plane tangent to the graph of f at the point $(1, -1, -5)$.