1. (40) Some basic integrals. (**Show all work**)

   (a) Calculate $\int_0^1 \int_x^1 e^{x/y}\, dy\, dx$ by first reversing the order of integration.

   (b) Use polar coordinates to evaluate $\int_0^{\sqrt{2}} \int_y^{\sqrt{4-y^2}} \frac{1}{1+x^2+y^2}\, dx\, dy$.

2. (20) Surface Integrals (**Show all work**)

(a) Let M be the portion of the cylinder given in cylindrical coordinates by

$$0 \leq z \leq 3, \qquad r = 1, \qquad 0 \leq \theta \leq \pi/2.$$

Orient M by normal vectors pointing away from the z-axis. Let C denote the boundary of M oriented counterclockwise when viewing M from the point $(5, 5, 1)$. Express the line integral $\oint_C \langle yz, -2xz, 0 \rangle \bullet d\mathbf{r}$ as a surface integral over M. **Do not evaluate**.

(b) Write down a parametrization $\Phi : D \to S$ of the surface S which is the graph of the function $x = y^2 + z^3$ over a domain D. Then determine the normal vector associated to this parametrization.

3. (30) Let M be the same surface as in (2a), namely the portion of the cylinder given in cylindrical coordinates by

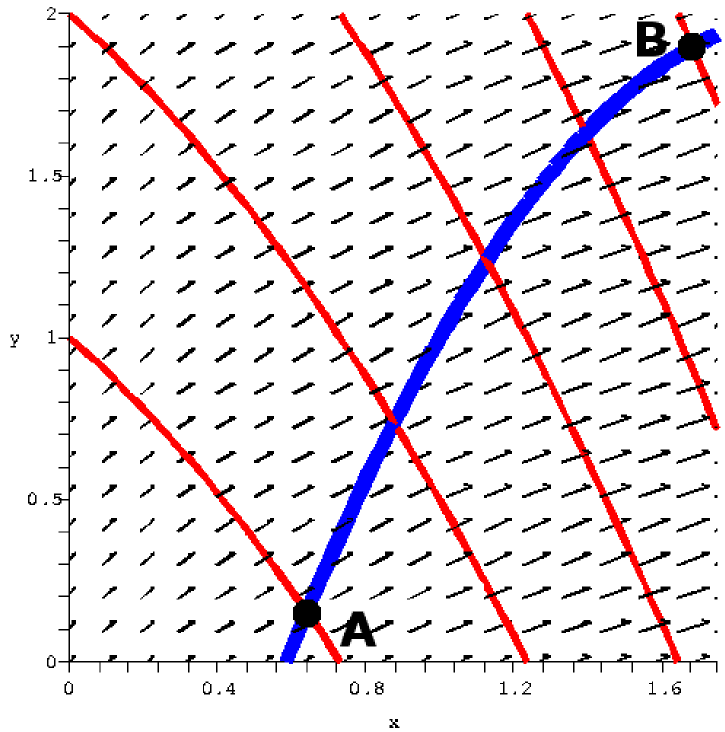
$$0 \leq z \leq 3, \qquad r = 1, \qquad 0 \leq \theta \leq \pi/2.$$

Orient M by normal vectors pointing away from the z-axis. By direct computation, calculate the flux (surface integral) of $\mathbf{F} = \langle 2x, y, -3z \rangle$ across M using the natural parametrization $\mathbf{\Phi}(\theta, z) = \langle \cos\theta, \sin\theta, z \rangle$.

4. (30) (**Show all work**) Let C denote the oriented closed curve consisting of the line segment from $(0,0)$ to $(\sqrt{2},0)$, followed by the arc of the circle $x^2+y^2=2$ from $(\sqrt{2},0)$ to $(1,1)$, followed by the line segment from $(1,1)$ to $(0,0)$. By any means you like, find the value of the line integral $I=\oint_C -y^3\,dx+x^3\,dy$.

5. (10) Short Answer. (Put answers in blanks provided; No partial credit)

   (a) The figure below shows a gradient vector field of a smooth function f and five level curves of f. The values of f on two adjacent level curves differs by 10 units. Consider the oriented curve C which goes from the point A to the point B. What is a good estimate of $\int_C \nabla f \bullet d\mathbf{r}$?



Answer: ______________________________

   (b) (**Show all work**) Consider the vector field $\mathbf{F} = \langle 3x + 2yz, 2x - y + z, x - 3y + 2z \rangle$ and the unit cube $([0,1] \times [0,1] \times [0,1])$ in the first octant. Find the flux of $\mathbf{F}$ out of the surface of this cube.

Answer: ______________________________

6. (20) **Multiple Choice** Circle the correct response. (No partial credit will be given)

(a) Suppose that $f(x, y)$ has continuous second partials on an open domain D, and that (a, b) is a critical point of f lying in D. Suppose that $f_{xx}(a, b) = -2$ and $f_{yy}(a, b) = 3$. What can be said about the critical point (a, b)?

A. nothing can be concluded from the information

B. (a, b) is a local minimum of f

C. (a, b) is a local maximum of f

D. (a, b) is a saddle point of f

E. none of the above

(b) For every smooth function f, the integral $\displaystyle\int_0^1 \int_0^{2y^2+1} f(x, y)\, dx\, dy$ is equal to

A. $\int_0^3 \int_0^{\sqrt{(x-1)/2}} f(x, y)\, dy\, dx$

B. $\int_1^3 \int_0^{\sqrt{(x-1)/2}} f(x, y)\, dy\, dx$

C. $\int_0^3 \int_{\sqrt{(x-1)/2}}^1 f(x, y)\, dy\, dx$

D. $\int_1^3 \int_{\sqrt{(x-1)/2}}^1 f(x, y)\, dy\, dx$

E. none of the above

(c) Let C be a curve from $(0,0)$ to $(2,1)$. According to the fundamental theorem for line integrals $\int_C (y-1)dx + (x+2y)dy$ is equal to:

A. 1 **B**. 2 **C**. 3 **D**. 4 **E**. It depends upon C

(d) If C is the boundary of a planar domain D, and C is oriented as in the statement of Green's theorem, then $\oint_C x^2y\,dx - y\,dy$ equals

A. $\iint_D (2xy-1)\,dA$ **B**. $\iint_D (1-x^2)\,dA$ **C**. $\iint_D (-x^2)\,dA$

D. none of the above

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Math 11

3 December 2005
Final Exam

Problem	Points	Score
1	40	
2	20	
3	30	
4	30	
5	10	
6	20	
Total	150	