Here are some problems to keep you awake at night. As usual these are not necessarily representative of the problems on the final, but should give you a decent review of the new material. You are on your own for the old material.

1. Consider two vector fields $\mathbf{F} = \langle -y, x \rangle$ and $\mathbf{G} = \langle \cos x + y, x - 1 \rangle$ defined in the plane.

   (a) Determine whether $\mathbf{F}$ or $\mathbf{G}$ is conservative. If conservative, produce a potential function.

   (b) Let C be the oriented curve from $(-3, 0)$ to $(1, 0)$ given as follows: the straight line from $(-3, 0)$ to $(-1, 0)$, then the clockwise arc of the unit circle to the point $(1, 0)$. Compute the line integrals $\int_C \mathbf{F} \bullet d\mathbf{r}$ and $\int_C \mathbf{G} \bullet d\mathbf{r}$.

2. Let M be the surface of the potato chip which is that part of the surface $z = xy$ inside the cylinder $x^2 + y^2 = 1$, and let C be its boundary positively oriented. If $\mathbf{F} = \langle 3xz - y, xz + yz, x^2 + y^2 \rangle$, find $\oint_C \mathbf{F} \bullet d\mathbf{r}$.

3. Let E denote the portion of the solid sphere of radius R in the first octant, and let $\mathbf{F} = \langle 2x+y, y^2, \cos(xy) \rangle$. Compute the flux of $\mathbf{F}$ (surface integral) across the boundary of E, oriented by the outward-pointing normal vectors.

4. Let C denote the circle of radius R centered at the origin and oriented counterclockwise. Let $\mathbf{F} = \langle \arctan x + y^3, 2x - \sqrt[3]{y} \rangle$. Compute $\oint_C \mathbf{F} \bullet d\mathbf{r}$.

5. Compute the flux of the vector field $\mathbf{F} = \langle x^3, 2xz^2, 3y^2 z \rangle$ over the surface M where M is the boundary of the solid bounded by the paraboloid $z = 4 - x^2 - y^2$ and the xy-plane.

6. Compute $\int_C y\,dx + x\,dy + (x^2 + y^2)\,dz$ where C is the positively oriented curve which bounds that part of the unit sphere in the first octant. Note that this is a closed curve consisting of three parts.