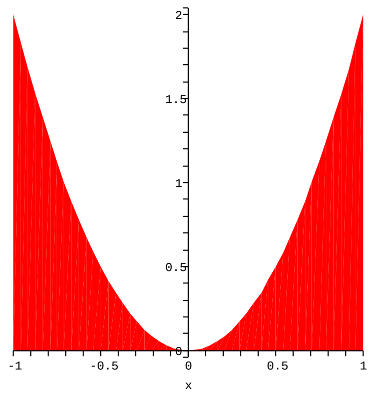
1. (25) (**Show all work**) The double integral $\iint_D y\, e^{x^5} dA = \iint_D y\, exp(x^5) dA$ can be expressed as the iterated integral $\int_{-1}^{1} \int_{0}^{2x^2} y\, e^{x^5}\, dydx$.

   (a) Evaluate this iterated integral.

   $$\int_{-1}^{1} \frac{y^2}{2}\Big|_0^{2x^2} e^{x^5}\, dx = \int_{-1}^{1} 2x^4 e^{x^5}\, dx = \frac{2}{5}e^{x^5}\Big|_{-1}^{1} = \frac{2}{5}(e - e^{-1})$$

   (b) Draw (and shade) the region D corresponding to $\iint_D y\, e^{x^5} dA$.



   (c) Express the double integral $\iint_D y\, e^{x^5} dA$ as iterated integral(s) in which the order of integration is reversed from part (a). **Do not evaluate.**

   $$\int_0^2 \int_{-1}^{-\sqrt{y/2}} f\, dxdy + \int_0^2 \int_{\sqrt{y/2}}^{1} f\, dxdy$$

2. (30) (**Show all work**) Consider the function $f(x,y) = xye^{x+2y}$.

   (a) Find all critical points of this function.

   $f_x = y(x+1)e^{x+2y}$ and $f_y = x(1+2y)e^{x+2y}$, and since the exponential function never vanishes, we have

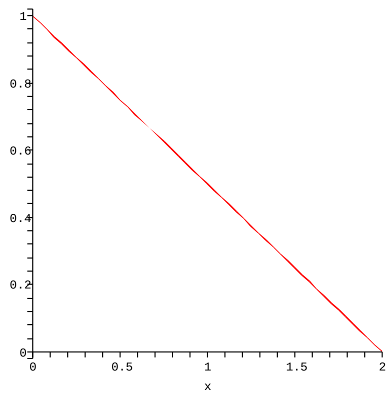
   $f_x = 0$ if and only if $y = 0$ or $x = -1$
   $f_y = 0$ if and only if $x = 0$ or $y = -1/2$

   This yields $(0,0)$ and $(-1,-1/2)$ as the critical points.

   (b) You may assume that a computation shows that the second partials of f are given by: $f_{xx} = y(2+x)e^{x+2y}$, $f_{xy} = (1+2y+x+2xy)e^{x+2y}$, $f_{yy} = 4x(1+y)e^{x+2y}$. Classify the above critical points.

Using the second derivative test, we find that at $(0,0)$, $D = \begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix} = -1 < 0$ so $(0,0)$ is a saddle point, while at $(-1,-1/2)$, $D = \begin{vmatrix} -e^{-2}/2 & 0 \\ 0 & -2e^{-2} \end{vmatrix} = e^{-4} > 0$ with $f_{xx}(-1,-1/2) < 0$ so $(-1,-1/2)$ is a local maximum.

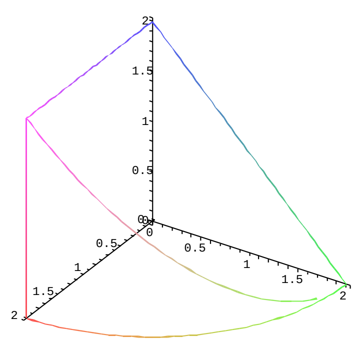
(c) Find the absolute maximum and minimum of f on the triangular region containing the points $(0,0)$, $(0,1)$, and $(2,0)$. Also indicate the points at which the absolute extrema occur.



Note that there are no critical points in the interior of the region, so the absolute extrema must occur on the boundary. On the two axes, f is identically zero, so we need only check the behavior on the line $y = -x/2 + 1$, $0 \le x \le 2$. Substituting into f, we obtain $g(x) = f(x, -x/2+1) = (\frac{-1}{2}x^2 + x)e^2$ on $[0,2]$. The endpoints correspond to points on the axes at which we already know the function is zero. $g'(x) = e^2(-x+1)$ which vanishes at $x = 1$. $g(1) = e^2/2$.

So the absolute minimum is zero occurring at all points on the axes and the absolute maximum is $e^2/2$ occuring at $(1, 1/2)$.

3. (15) (**Show all work**) Let W be the wedge (depicted below) in the first octant bounded by $x^2+y^2 = 4$ and $y+z = 2$. Let f be a continuous function on W and set $I = \iiint_W f\, dV$.



(a) Express the triple integral I as an iterated integral $\iiint f\,dz\,dx\,dy$.

$\int_0^2 \int_0^{\sqrt{4-y^2}} \int_0^{2-y} f\, dzdxdy$

(b) Express the triple integral I as an iterated integral $\iiint f\,dx\,dy\,dz$.

$\int_0^2 \int_0^{2-z} \int_0^{\sqrt{4-y^2}} f\, dxdydz$

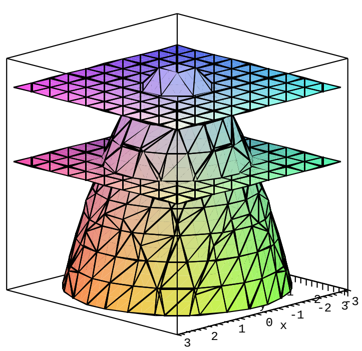
(c) Express the triple integral $\iiint_W (x+y+z)\,dV$ in cylindrical coordinates. **Do not evaluate.**

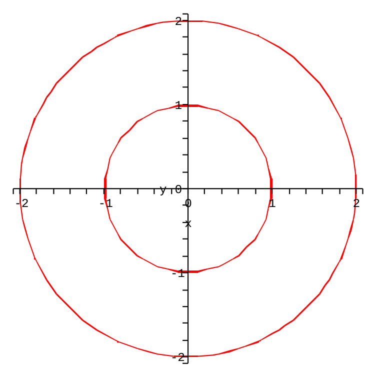
$\int_0^{\pi/2} \int_0^2 \int_0^{2-r\sin(\theta)} (r\cos(\theta) + r\sin(\theta) + z) r\, dzdrd\theta$

4. (15) (**Show all work**) Evaluate $\displaystyle\iint_D e^{x^2+y^2}\,dA$ where D is the region in the third quadrant bounded by $x^2+y^2=7$.

$$\int_{\pi}^{3\pi/2} \int_0^{\sqrt{7}} e^{r^2} r\, drd\theta = \int_{\pi}^{3\pi/2} \left(\frac{e^{r^2}}{2}\Big|_0^{\sqrt{7}} \right) d\theta = \pi(e^7-1)/4.$$

5. (15) (**Show all work**) Compute the surface area of that part of the paraboloid $z = 9 - x^2 - y^2$ which is between the planes $z=5$ and $z=8$.





The paraboloid together with the two planes are drawn on the left. The projection into the xy-plane of that part of the surface trapped between the planes is pictured on the right as the area between the two circles of radius 1 and 2 respectively (an annulus). Denote by D that annulus. Let $f(x,y) = z = 9 - x^2 - y^2$.

The surface area equals $\displaystyle\iint_D \sqrt{1 + f_x^2 + f_y^2}\,dA = \iint_D \sqrt{1+4x^2+4y^2}\,dA$. Converting to polar coordinates, we obtain

$$\iint_D \sqrt{1+4x^2+4y^2}\,dA = \int_0^{2\pi} \int_1^2 \sqrt{1+4r^2}\; r\,drd\theta = \int_0^{2\pi} \frac{1}{12}(1+4r^2)^{3/2}\Big|_1^2\, d\theta =$$

$2\pi\frac{1}{12}(17^{3/2} - 5^{3/2}) = \frac{\pi}{6}(17^{3/2} - 5^{3/2})$.