NOTE: In no sense should this collection of problems be construed as representative of the actual exam. These are simply some problems left over from our preparation of the exam or from previous exams which should serve to indicate the general level of expectation.

Ok; here are some supposed solutions ... no guarantees that there are no typos!

1. Evaluate the double integral $\iint_D xy^2\,dA$ if D is the region in the half-plane $x \geq 0$ bounded by the curves $x = 1 - y^2$ and $x^2 + y^2 = 1$.

   It is easiest to do this in the order $\iint xy^2\;dxdy$:

   $\iint_D xy^2\,dA = \int_{-1}^{1}\int_{1-y^2}^{\sqrt{1-y^2}} xy^2\,dx\,dy = \frac{1}{2}\int_{-1}^{1} y^2(1-y^2-(1-y^2)^2)\,dy = \frac{2}{35}$

2. Let E be the region in the first octant bounded by the surfaces $2y^2 + z^2 = 8$ and $x + y = 2$ and let f be a continuous function whose domain includes E. Denote by $I = \iiint_E f dV$.

   (a) Set up an iterated integral equal to I of the form $\iiint f\,dz\,dx\,dy$

   The integration is over the region in the first octant above the triangle in the xy-plane bounded by the coordinate axes and the line $x + y = 2$, and below the graph of $z = \sqrt{8 - 2y^2}$.

   $I = \int_0^2 \int_0^{2-y} \int_0^{\sqrt{8-2y^2}} f\,dz\,dx\,dy.$

   (b) Set up an iterated integral equal to I of the form $\iiint f\,dx\,dy\,dz$

   The projection of the region onto the yz-plane is the region in the first quadrant of that plane and inside the ellipse $2y^2 + z^2 = 8$.

   $I = \int_0^{2\sqrt{2}} \int_0^{\sqrt{(8-z^2)/2}} \int_0^{2-y} f\,dx\,dy\,dz.$

3. Find and classify all critical points of the function $f(x, y) = 5x^2y - 2xy^2 + 30xy - 3$.

   The gradient of f is $\nabla f = \langle 10xy - 2y^2 + 30y, 5x^2 - 4xy + 30x\rangle$.

   $\nabla f = \mathbf{0}$ if and only if $2y(5x - y + 15) = 0$ and $x(5x - 4y + 30) = 0$, so we see that the critical points are where:

   $f_x = 0$: $y = 0$ or $5x - y + 15 = 0$ and

   $f_y = 0$: $x = 0$ or $5x - 4y + 30 = 0$.

   We obtain four critical points: $(0, 0), (6, 0), (0, 15), (-2, 5)$

   The second derivative test indicates the first three are saddle points, while the last is a local minimum.

4. Sketch the region of integration and evaluate the integral $\int_0^2 \int_{2y}^4 \sin(x^2)\,dx\,dy$ by changing the order of integration.

   You draw it! I'm typing!

   $\int_0^2 \int_{2y}^4 \sin(x^2)\,dx\,dy = \int_0^4 \int_0^{x/2} \sin(x^2)\,dy\,dx = \int_0^4 \frac{x}{2}\sin(x^2)\,dx = -\frac{1}{4}\cos(x^2)\;\Big|_0^4 = \frac{1-\cos(16)}{4}$

5. Integrate $\iint_D y^2\, dA$ where D is the region bounded by $x^2 + y^2 = 4$ and $y \geq 0$.

   This seems ripe for polar coordinates:

   $\iint_D y^2\, dA = \int_0^\pi \int_0^2 (r \sin\theta)^2 r\, dr d\theta = \int_0^\pi \sin^2\theta\, d\theta \int_0^2 r^3\, dr = 2\pi$

6. Consider the triple iterated integral $\int_0^3 \int_{2z/3}^2 \int_0^{4-y^2} y\, dx\, dy\, dz$. Rewrite this integral as $\iiint y\, dz\, dx\, dy$

   The region of integration is part of the first octant bounded by the coordinate planes, the plane $y = 2z/3$ and the cylinder $x = 4 - y^2$.

   $\int_0^3 \int_{2z/3}^2 \int_0^{4-y^2} y\, dx\, dy\, dz = \int_0^2 \int_0^{4-y^2} \int_0^{3y/2} y\, dz\, dx\, dy.$