1. (20) (**Show all work**) Once again the starship Puddlejumper finds itself in trouble. With engines dead, they have drifted near the sun and find themselves watching their hull temperature which changes according to the function $T(x, y, z) = 1000e^{x^2-4y^2-9z^2}$ degrees Kelvin. They are at the position $(2, 1, 0)$ when their engineer gets the engines back online.

   (a) In which direction (unit vector) should they proceed to cool the ship most rapidly?

   (b) At what rate (degrees/unit distance) will it cool if they go in that direction?

   (c) Sadly, the Puddlejumper is a sorry old ship and the hull will breach if cooled faster than $500\sqrt{80}$ degrees per unit distance. If θ is the angle between the direction of **maximum increase** of T and the unit vector $\mathbf{u}$ defining their desired direction of travel to cool the ship, what constraints exist, if any, on the angle θ?

2. (20) (**Show all work**) The surface of a hill is described by the graph of $z = 100 - x^2 - y^2$; z is the height above the flat ground in meters. Suppose that a model plane flying above the hill spirals down following the curve $\mathbf{r}(t) = \langle 2\sin t, 2\cos t, 200 - 4t\rangle$.

   (a) At what time t does the model plane hit the hill?

   (b) At what point (x, y, z) does the crash occur?

   (c) What is the length of the flight path of the model plane starting from $t = 0$ to the time of impact?

   (d) Supposing now that the airplane is flying above level ground (the mountain has been moved out of the way!), it is clear that it strikes the ground at $t = 50$. Find the angle of impact with the ground, that is the angle between the flight path as the plane strikes and the ground.

3. (10) (**Show all work**) Consider the surface $x^2 + y^2 + z^2 = 9$. Find the point of intersection of the tangent plane to the surface at the point $(1, 2, 2)$ and the x-axis.

4. (10) (**Show all work**) Suppose that $F(x, y)$ is differentiable on all of $\mathbb{R}^2$, and define $g(s, t) = F(s^2 - t^2, t^2 - s^2)$. Show that g satisfies the partial differential equation

$$t\frac{\partial g}{\partial s} + s\frac{\partial g}{\partial t} = 0.$$

5. (40) **Multiple Choice** Circle the correct response. (No partial credit will be given)

(a) If the motion of a particle is given by a vector valued function $\mathbf{r}(t)$ defined for $a \leq t \leq b$, then the integral of the speed of $\mathbf{r}(t)$ from $t = a$ to $t = b$ equals

A. the acceleration **B**. the distance from $\mathbf{r}(a)$ to $\mathbf{r}(b)$

C. the velocity **D**. the distance the particle travels from $\mathbf{r}(a)$ to $\mathbf{r}(b)$

E. none of the above

(b) The value of the limit $\displaystyle\lim_{(x,y)\to(0,0)} \frac{\sin(xy)}{x^2+y^2}$ is

A. 0 **B**. 1 **C**. 1/2 **D**. -1/2 **E**. none of the above

(c) If $f(x,y)$ is continuous at (a,b) and $\lim\limits_{(x,y)\to(a,b)} f(x,y) = 7$ along every straight line through the (a,b), then $\lim\limits_{(x,y)\to(a,b)} f(x,y) =$

A. cannot be determined **B**. 7

C. 7 only if f is differentiable at (a,b)

D. depends on the value of $f(a,b)$ **E**. none of the above

(d) If $f(x,y) = \int_x^y \cos(t^3)\,dt$, then $\dfrac{\partial f}{\partial x} =$

A. $\cos(x^3)$ **B**. $-3x^2\cos(x^3)$ **C**. $\sin(x^3)$

D. $-3x^2\sin(x^3)$ **E**. none of the above

(e) Suppose that you are given a function $f(x,y)$ and vectors $\mathbf{u} = \langle \frac{1}{2}, \frac{-\sqrt{3}}{2} \rangle$ and $\mathbf{v} = \langle \frac{1}{2}, \frac{\sqrt{3}}{2} \rangle$. If $(D_{\mathbf{u}} f)(x_0, y_0) = 2$ and $(D_{\mathbf{v}} f)(x_0, y_0) = -1$, then $\dfrac{\partial f}{\partial x}(x_0, y_0) =$

A. $\frac{1}{2}$ **B**. 1 **C**. 2 **D**. $\sqrt{3}$ **E**. none of the above

(f) Suppose that f is a function of the variables x, y, and z, and that x is a function of the variables s and t, y is a function only of s and z is a function only of the variable t. What is the correct expression for $\dfrac{\partial f}{\partial t}$?

A. $\dfrac{\partial f}{\partial x} + \dfrac{\partial f}{\partial y} + \dfrac{\partial f}{\partial z}$ **B**. $\left(\dfrac{\partial f}{\partial x} + \dfrac{\partial f}{\partial y} + \dfrac{\partial f}{\partial z} \right) \dfrac{\partial f}{\partial t}$

C. $\dfrac{\partial f}{\partial x}\dfrac{\partial x}{\partial t} + \dfrac{\partial f}{\partial y}\dfrac{\partial y}{\partial s} + \dfrac{\partial f}{\partial z}\dfrac{\partial z}{\partial t}$ **D**. $\dfrac{\partial f}{\partial x}\dfrac{\partial x}{\partial t} + \dfrac{\partial f}{\partial z}\dfrac{\partial z}{\partial t}$

E. none of the above

(g) Suppose that $w = f(x, y, z)$ is differentiable in a neighborhood of the point $(7, 8, 9)$ and that $f(7, 8, 9) = -2$, $f_x(7, 8, 9) = 3$, $f_y(7, 8, 9) = 5$, and $f_z(7, 8, 9) = 7$. What can be said about the change in f at the point $(7, 8, 9)$ in the direction indicated by the vector $\langle -1, 2, -1 \rangle$?

A. strictly increasing **B**. strictly decreasing **C**. unchanging

D. first increasing, then decreasing **E**. impossible to be determined

(h) Suppose that the graph of $z = f(x, y)$ represents the surface of a mountain, and you are standing at a point (x_0, y_0, z_0) on the surface. You are told that the gradient of f at (x_0, y_0) is $\nabla f(x_0, y_0) = \langle 1, 3 \rangle$. If you move in the direction of the gradient, what is your initial angle of elevation?

A. $\tan^{-1} 3$ **B**. $\cos^{-1} 3$ **C**. $\tan^{-1} \sqrt{10}$ **D**. $\cos^{-1} \sqrt{10}$

E. none of the above

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Math 11

17 October 2005
Hour Exam I

Problem	Points	Score
1	20	
2	20	
3	10	
4	10	
5	40	
Total	100	