

Math 11. Multivariable Calculus.

Practice Homework 9.

No due date.

This homework set is not to be turned in. It is for you to practice and prepare for the exam. Solutions will be posted on the course website.

1. Find the center of mass of the hemisphere $x^2 + y^2 + z^2 = a^2$, $z \geq 0$, if it has constant density.
2. A particle moves along line segments from the origin to the points $(1, 0, 0)$, $(1, 2, 1)$, $(0, 2, 1)$, and then back to the origin under the influence of the force field $\mathbf{F} = \langle z^2, 2xy, 4y^2 \rangle$. Find the work done in two separate ways: (a) by directly calculating this line integral, and (b) by using Stokes' Theorem with a suitable choice of surface S .
3. Let $\mathbf{F} = \langle z \tan^{-1}(y^2), z^3 \ln(x^2 + 1), z \rangle$. Find the flux of \mathbf{F} across the part of the paraboloid $x^2 + y^2 + z = 2$ that lies above the plane $z = 1$ and is oriented upwards.
4. Use the Divergence Theorem to evaluate

$$\iint_S (2x + 2y + z^2) dS,$$

where S is the sphere $x^2 + y^2 + z^2 = 1$.