Math 11. Multivariable Calculus.
Written Homework 8.
Due on Wednesday, 11/12/14.

You can turn in this homework by leaving it in the boxes labeled Math 11 in the hallway outside of 008 Kemeny anytime before 3:00 pm on Wednesday.

1. Let D be a region bounded by a simple closed path C in the xy-plane. Use Green's Theorem to prove that the coordinates $(\bar{x}, \bar{y})$ of the centroid (the centroid is the center of mass of D, if we assume that D is a lamina of uniform density ρ and area A) of D are
$$\bar{x} = \frac{1}{2A}\int_C x^2\,dy, \quad \bar{y} = -\frac{1}{2A}\int_C y^2\,dx.$$

2. Let C be the arc of the curve $y = \cos x$ from $(-\pi/2, 0)$ to $(\pi/2, 0)$, and $\mathbf{F} = \langle x + y^2, e^{-y^2} + x^2\rangle$ a vector field. Compute $\int_C \mathbf{F}\cdot d\mathbf{r}$.
Hint: Green's theorem may be helful here, if you find a way to turn C into a closed path.

3. Prove $\operatorname{div}(f\mathbf{F}) = f \operatorname{div}\mathbf{F} + \mathbf{F}\cdot\nabla f$ assuming that the appropriate partial derivatives exist and are continuous. Here $\mathbf{F} = \langle P, Q, R\rangle$ and P, Q, R, f are all scalar-valued functions of the variables x, y, z.

4. Find the surface area of the part of the cone $z = \sqrt{x^2 + y^2}$ that lies between the plane $y = x$ and the parabolic cylinder $y = x^2$.